Ver 21.00 – Base Motion **2/17/2020**

A steady state harmonic base excitation (motion) can be specified in this Ver 21.00. The bearings with station J = 0, with the exception of Floating Ring Bearing (see example 5 for more details), are connected to the base and the flexible supports with the non-zero stiffness and damping are also considered to be connected to the base, as shown in the Figures 1 and 2, and all the stations connected to the base are subject to the base excitation if specified. i.e., all the rotor/support stations connected to the ground are now considered to be connected to the base if base excitation is present. The foundation is neglected in the base motion analysis.

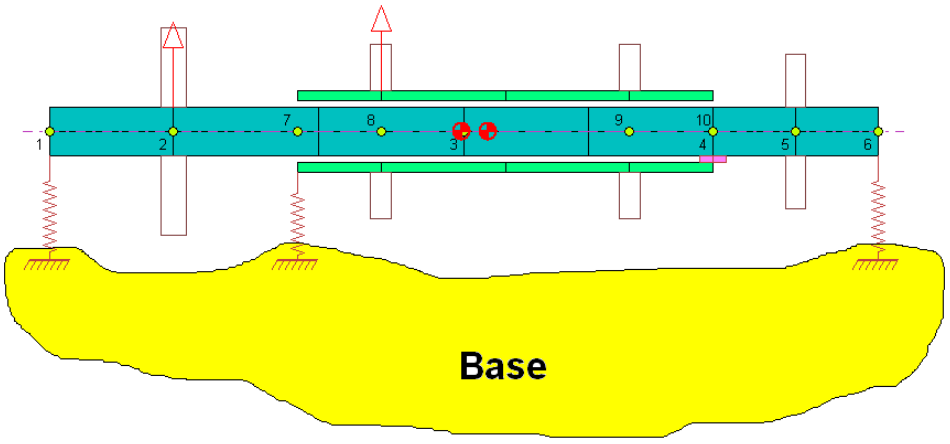


Fig. 1 – Single Base

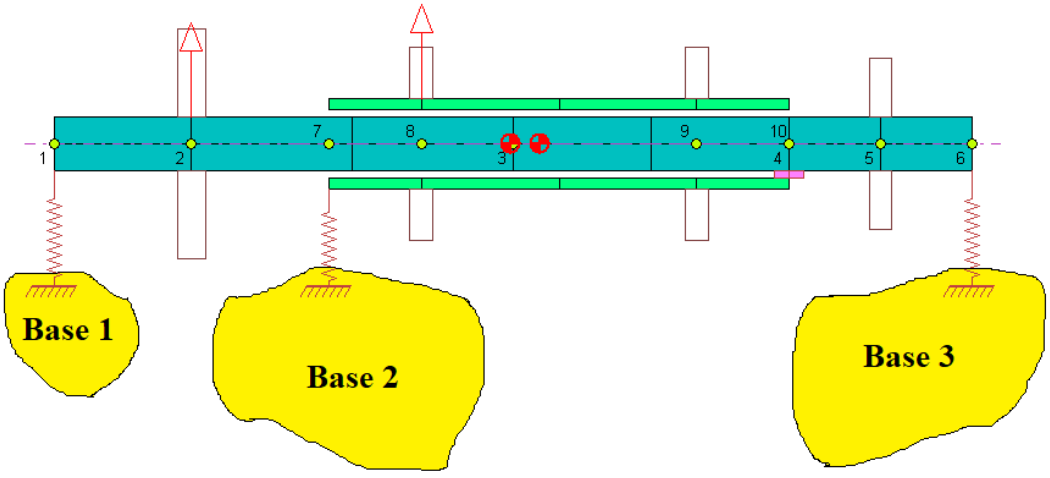


Fig. 2 – Multiple Bases

For the steady state harmonic base excitation analysis, the system must be linear and bearings are linear bearings. This is a linear analysis.

The base motion inputs are entered in the Base Motion tab under Model – Data Editor, as shown in Figure 3. The inputs are described below:

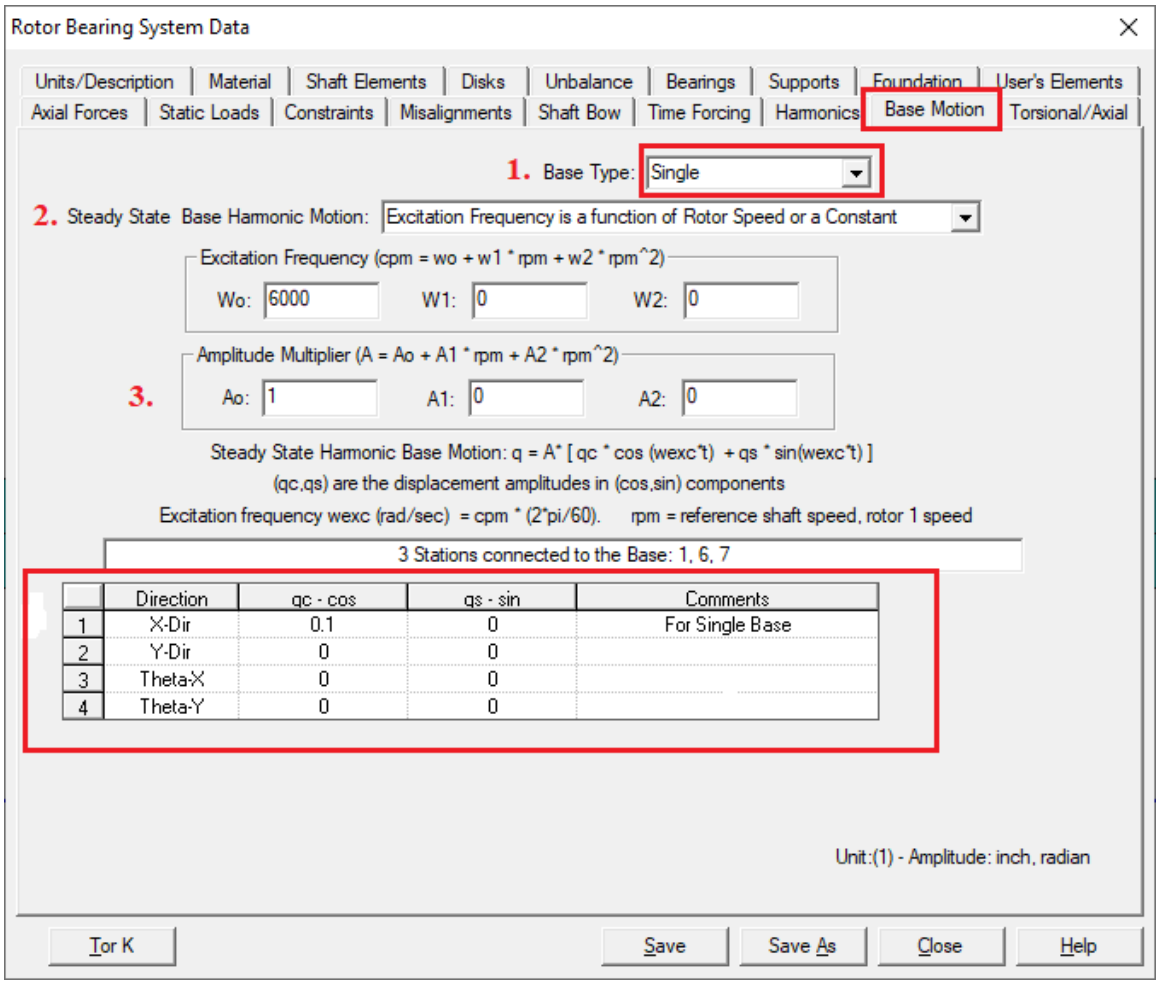
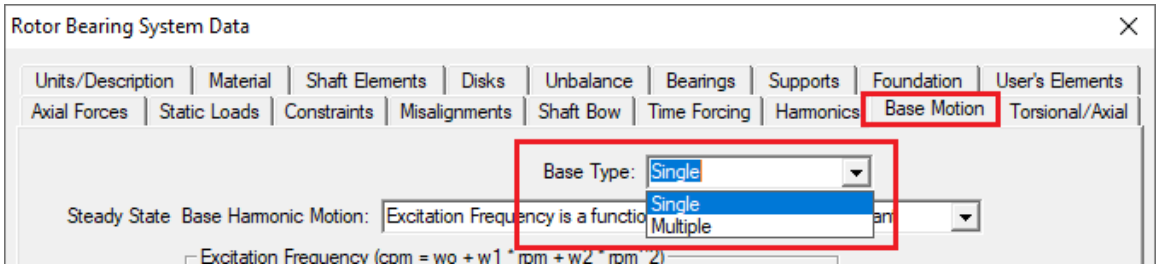


Fig. 3 – Inputs for Base Motion (Single Base)

1) Base Type: The Base Type can be Single, as shown in Fig. 1, or Multiple as shown in Fig. 2.



For a Single Base model, all the stations connected to the same base are subject to the same base motion. For a Multiple Bases model, the stations connected to the different base can have different base motion in amplitude and phase, but the base excitation

frequency is the same for all the bases, only the amplitude and phase can be different. Each base has 4 degrees-of-freedom, as described in the lateral vibration model, i.e., two translations (X, Y) and two rations (Theta-X and Theta-Y). For a Single Base, the inputs are illustrated in Fig. 3. Fig. 4 shows a multiple bases model input. There is a "station reminder" automatically shown above the motion input. It shows the number of stations connected to the base and the station numbers. For a multiple bases model, the base motion for all the connected stations must be entered.

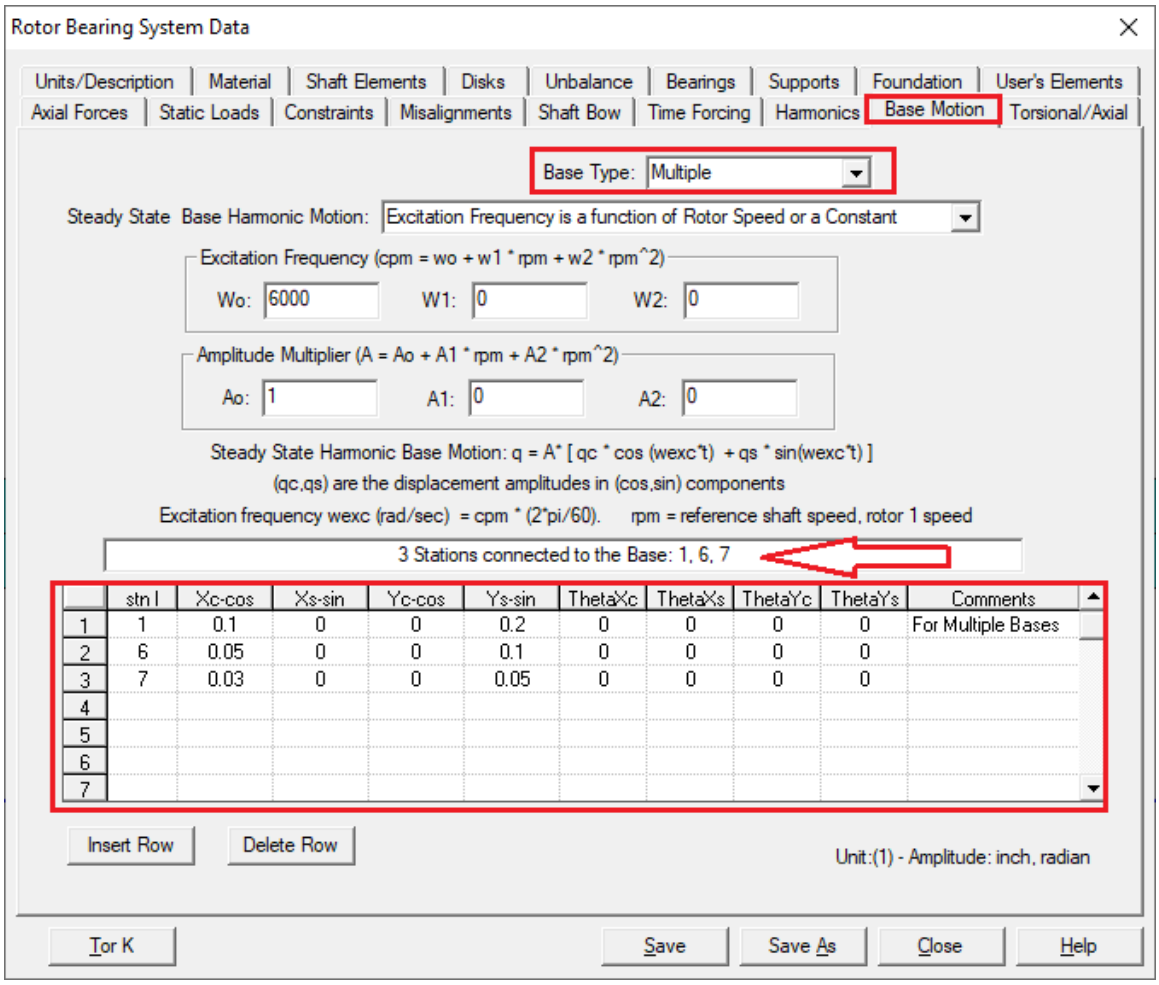
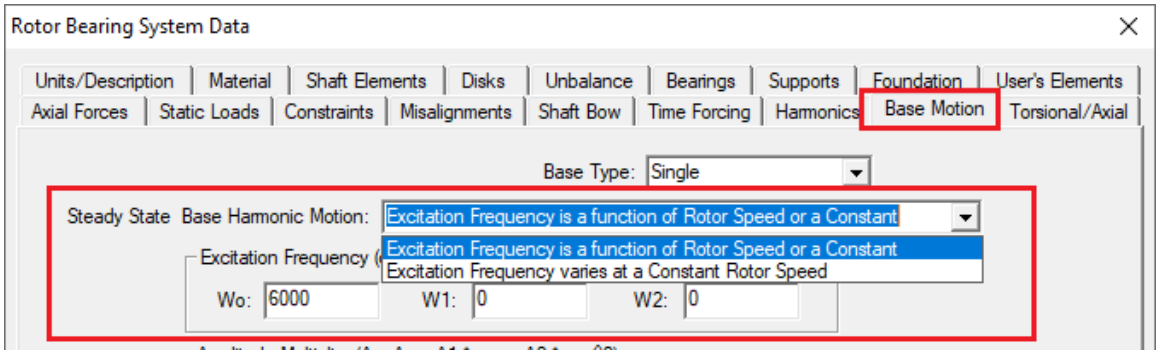


Fig. 4 – Inputs for Base Motion (Multiple Bases)

Note that if the rotational displacements are specified in the base motion, the bearings connected to the base must have rotational stiffness and/or damping to transmit the base motion.

2) Excitation Frequency: The base motion frequency (excitation frequency) can be either a function of rotor speed or a constant frequency, or the excitation frequency varies at a constant rotor speed.



If the excitation frequency is a function of rotor speed or a constant, the analysis is performed for a range of rotor speed, as illustrated in Fig. 5. If the excitation frequency varies at a constant rotor speed, the analysis is performed for a range of excitation frequency, as illustrated in Fig. 6.

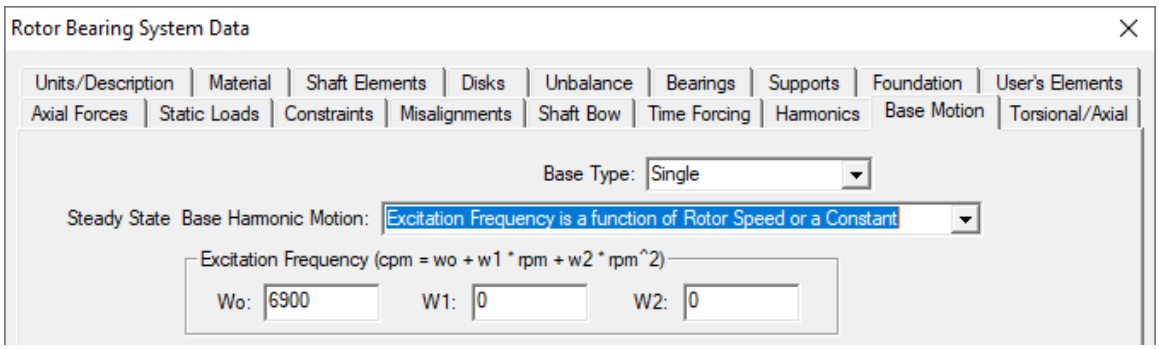


Fig. 5 – Excitation frequency is a constant

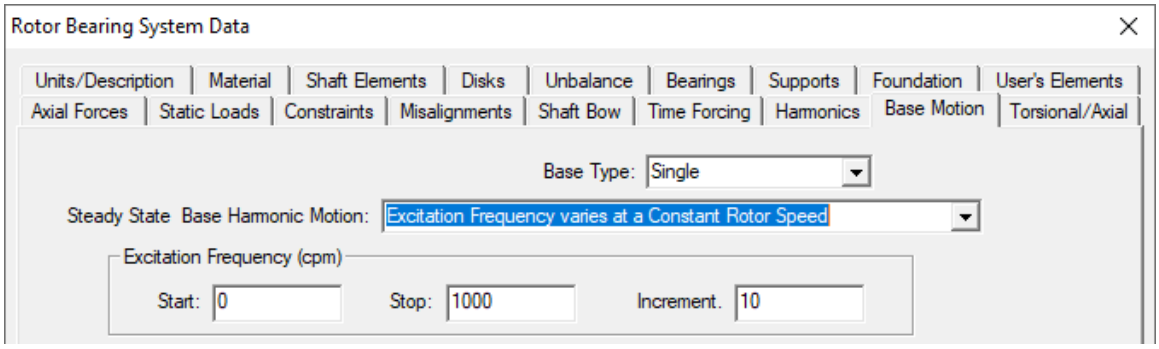


Fig. 6 – Excitation frequency varies at a constant rotor speed

3) Base Motion: The base motion is described as a steady state harmonic motion.

For the i^{th} base:

$$z_i = A\times\left[z_{ci}\cos\left(\omega_{exc}t\right)+z_{si}\sin\left(\omega_{exc}t\right)\right] \quad (1)$$

$$= A\times|z|\cos\left(\omega_{exc}t-\phi\right)$$

Note that the displacement expression uses a phase lag ($-\phi$), and the force expression uses a phase lead.

where A is a speed dependent amplitude multiple. In general, the base motion is speed independent, therefore A_o=1, A_1=A_2 = 0. For a multiple base model, the base excitation frequency is the same, but the amplitude and phase can be different by specifying different cosine and sine components (z_c and z_s) of the motion. For every base motion, 4 degrees-of-freedom can be specified: two translations (x,y) and two rotations (θ_x, θ_y). Since the motion is transmitted through bearings to the rotor system. If rotational base motion is specified (θ_x, θ_y), then the bearing connected to this base must have the rotational stiffness and/or damping to transmit this base motion to the rotor system. Otherwise, the rotational base motion will be ignored.

4) Analysis: When performing the base motion analysis, analysis option 23, the rotor speed input depends on the excitation frequency type entered in the Base Motion Input. As said before, if the excitation frequency is a function of rotor speed or a constant, the analysis can be performed in a range of rotor speed, and if the excitation frequency varies at a constant rotor speed, the analysis is performed at a constant rotor speed, as illustrated in Fig. 7.

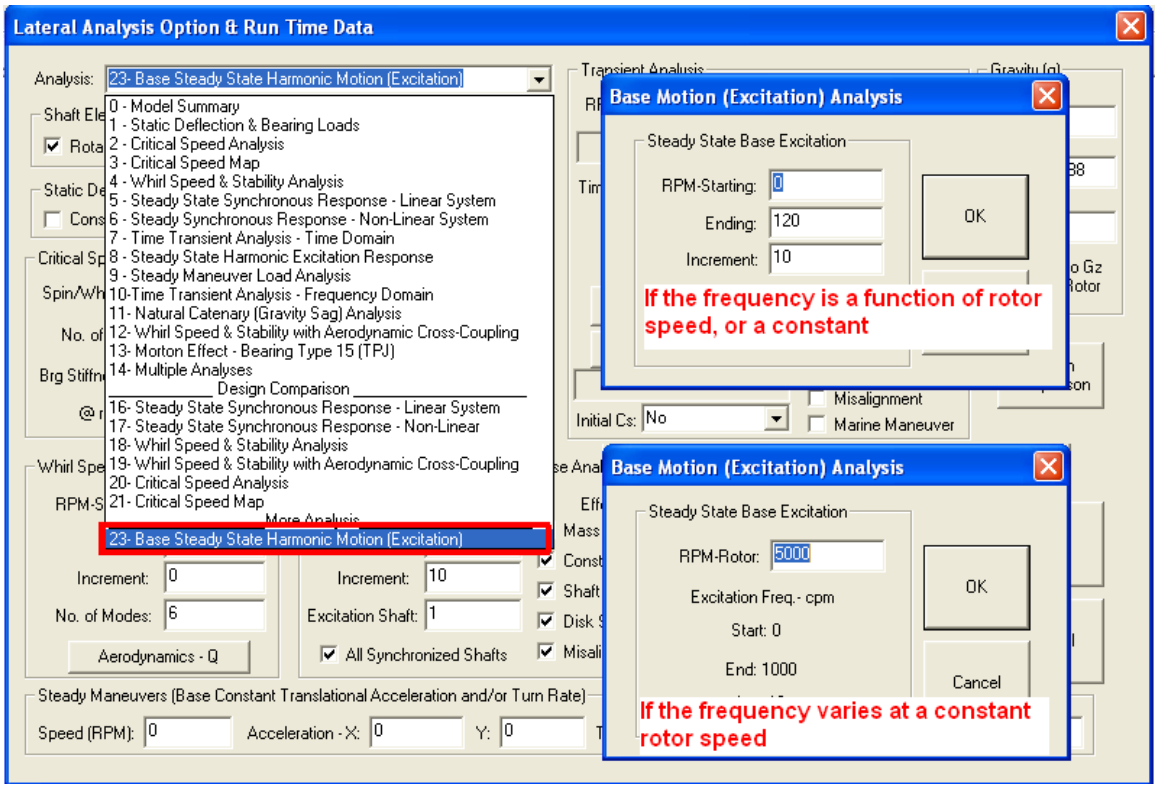
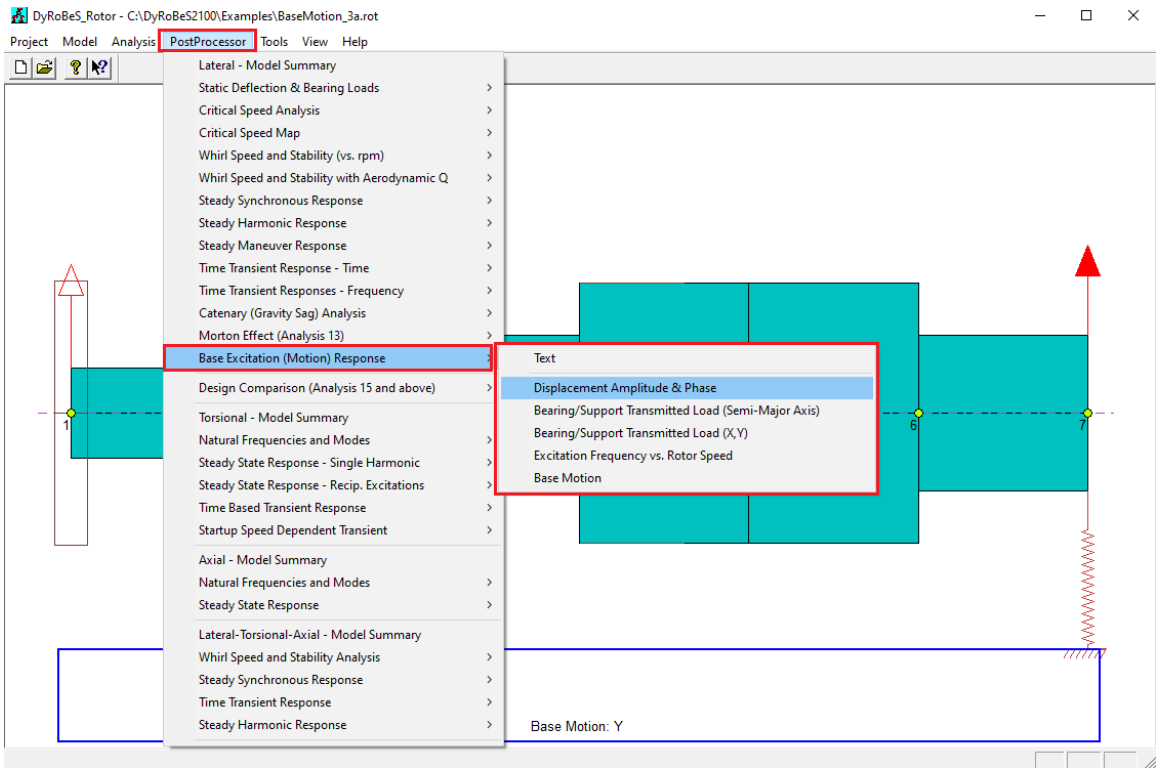


Fig. 7 – Rotor speed input for the base motion analysis

The results can be viewed from the Postprocessor in both text and graphic formats. Several examples are employed to illustrate the use of Base Motion Analysis. Mathematically, the absolute displacement of the rotor due to the base motion can be verified using the Steady State harmonic Excitation Analysis. It will be demonstrated in the following examples.



Example 1 – Single Degree-of-Freedom

For a single DOF system, a mass m is supported by a spring k and a damping c. The spring and damping connected to the base are subject to a base motion z(t).

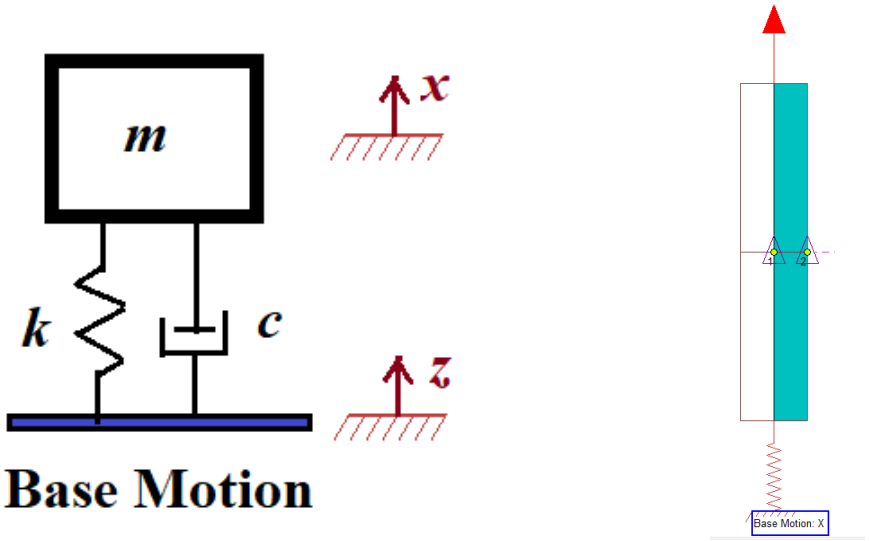


Fig. 8 – SDOF model

The equation of motion of the mass can be obtained by applying the Newton's 2nd law for the absolute displacement x is:

$$m\ddot{x} = F_k + F_c \qquad (2)$$

where F_k and F_c are the spring and damping forces acting on the mass m.

$$F_k = -k\left(x - z\right) \tag{3}$$

$$F_c = -c\left(\dot{x} - \dot{z}\right) \tag{4}$$

i.e.,

$$m\ddot{x} = -k\left(x - z\right) - c\left(\dot{x} - \dot{z}\right) \tag{5}$$

or

$$m\ddot{x} + c\dot{x} + kx = kz + c\dot{z} \tag{6}$$

For a harmonic base motion

$$z = z_c \cos\left(\omega_{exc} t\right) + z_s \sin\left(\omega_{exc} t\right) \tag{7}$$

Therefore, the equation of motion becomes:

$$m\ddot{x} + c\dot{x} + kx = \left(kz_c + c\omega_{exc} z_s\right)\cos\omega_{exc} t + \left(kz_s - c\omega_{exc} z_c\right)\sin\omega_{exc} t \tag{8}$$

Define the relative displacements with respect to the base motion z:

$$u = x - z \qquad \Rightarrow \qquad x = u + z \tag{9}$$

Substitution of Eq. (9) into Eq. (6), the equation of motion in the relative displacement form:

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{z} \tag{10}$$

<u>Case 1</u>: BaseMotion_1a.rot

The first case is taken from "Applied Mechanical Vibrations" by David V. Hutton, page 84.

m = 8 Lb, k = 40 Lb/in, c = 0, z_c = 0, z_s = 0.2 in, and
ω_{exc} = 115 Hz (6900 cpm = 722.57 rad/sec)

The system undamped natural frequency is:

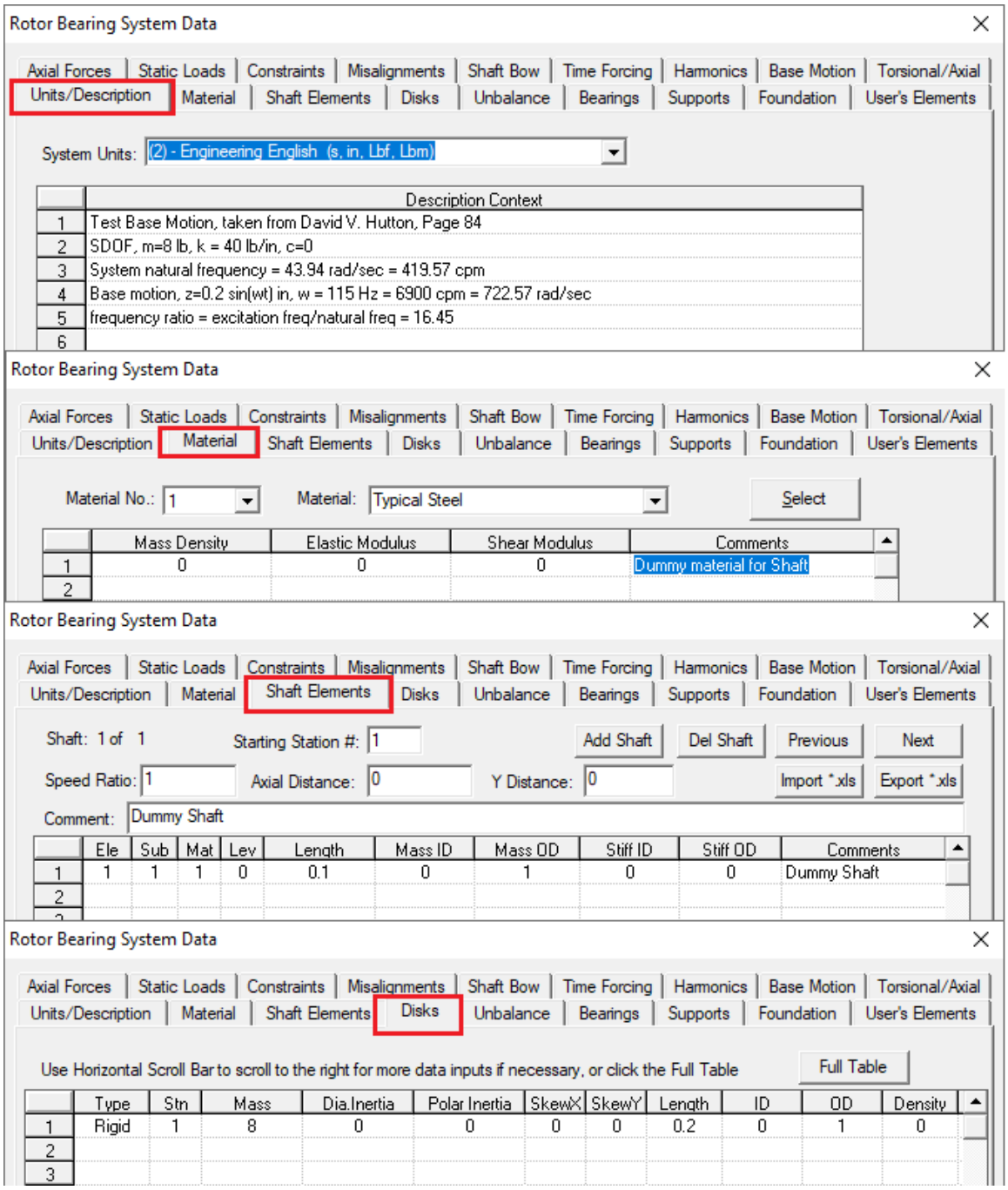
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{8/386.088}} = 43.94 \text{ rad/sec} = 419.57 \text{ cpm}$$

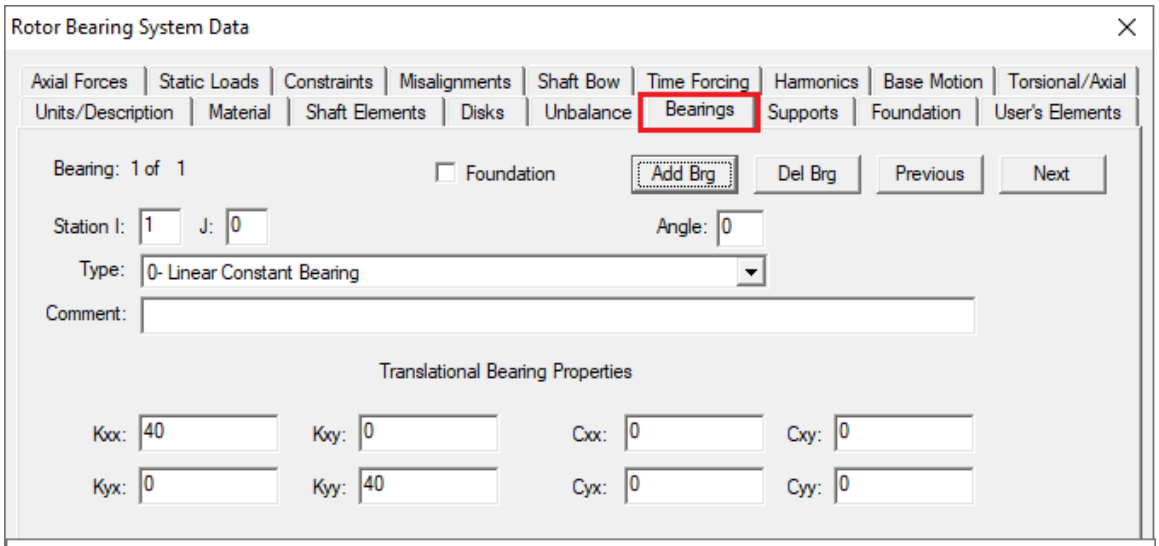
The frequency ratio is defined as below

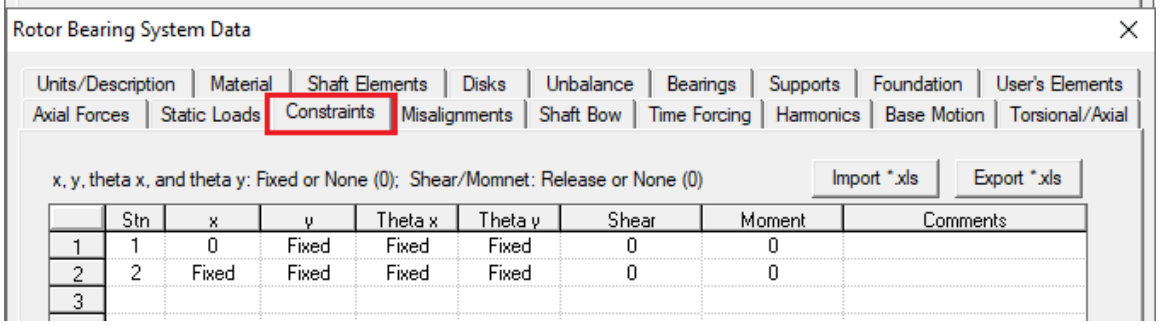
$$\gamma = \frac{\omega_{exc}}{\omega_n} = 16.45$$

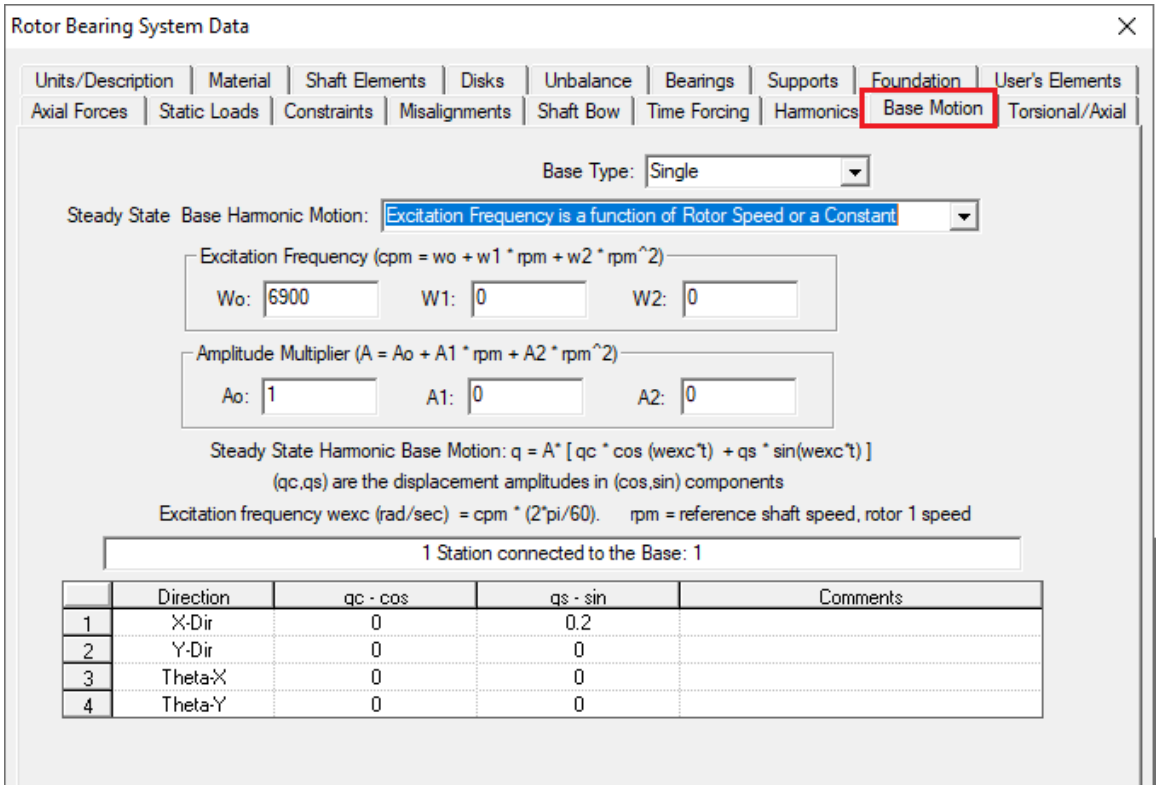
For this high frequency ratio, i.e., the excitation frequency is much higher than the system natural frequency, the inertia of the mass keeps it from moving much, so that the relative motion consists primarily of the base motion relative to the mass. The mass steady state vibration amplitude is 0.00074 in. The mass relative displacement to the base is 0.20074 in.

The related rotor-bearing model and base motion inputs and analysis inputs are shown below:




Rotor Bearing System Data
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Bearing: 1 of 1
Foundation
Add Brg
Del Brg
Previous
Next
Station I: 1
J: 0
Angle: 0
Type: 0- Linear Constant Bearing
Comment:
Translational Bearing Properties
Kxx: 40
Kxy: 0
Cxx: 0
Cxy: 0
Kyx: 0
Kyy: 40
Cyx: 0
Cyy: 0


Rotor Bearing System Data
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
x, y, theta x, and theta y: Fixed or None (0); Shear/Momnet: Release or None (0)
Import *.xls
Export *.xls
Stn
x
y
Theta x
Theta y
Shear
Moment
Comments
1 1 0 Fixed Fixed Fixed 0 0
2 2 Fixed Fixed Fixed Fixed 0 0
3


Rotor Bearing System Data
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Base Type: Single
Steady State Base Harmonic Motion: Excitation Frequency is a function of Rotor Speed or a Constant
Excitation Frequency (cpm = wo + w1 * rpm + w2 * rpm^2)
Wo: 6900
W1: 0
W2: 0
Amplitude Multiplier (A = Ao + A1 * rpm + A2 * rpm^2)
Ao: 1
A1: 0
A2: 0
Steady State Harmonic Base Motion: q = A* [qc * cos (wexc*t) + qs * sin(wexc*t)]
(qc,qs) are the displacement amplitudes in (cos,sin) components
Excitation frequency wexc (rad/sec) = cpm * (2*pi/60). rpm = reference shaft speed, rotor 1 speed
1 Station connected to the Base: 1
Direction
qc - cos
qs - sin
Comments
1 X-Dir 0 0.2
2 Y-Dir 0 0
3 Theta-X 0 0
4 Theta-Y 0 0

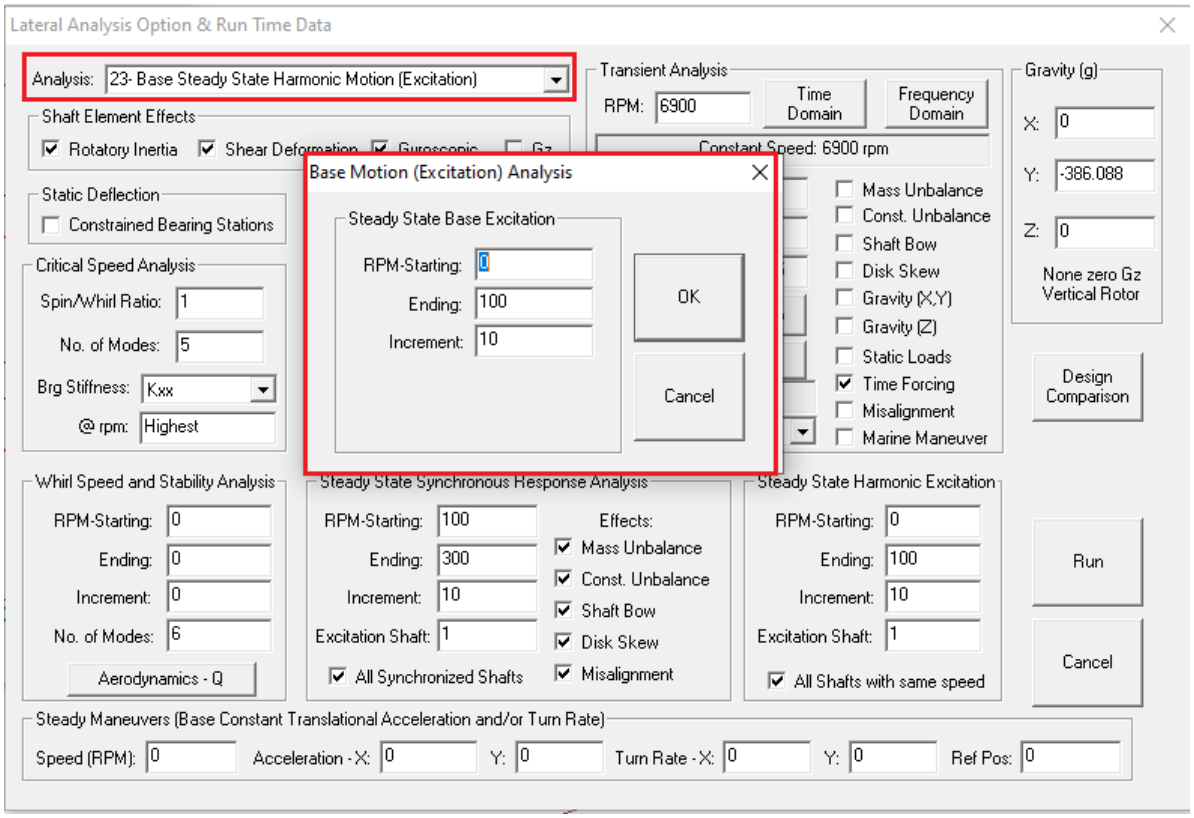
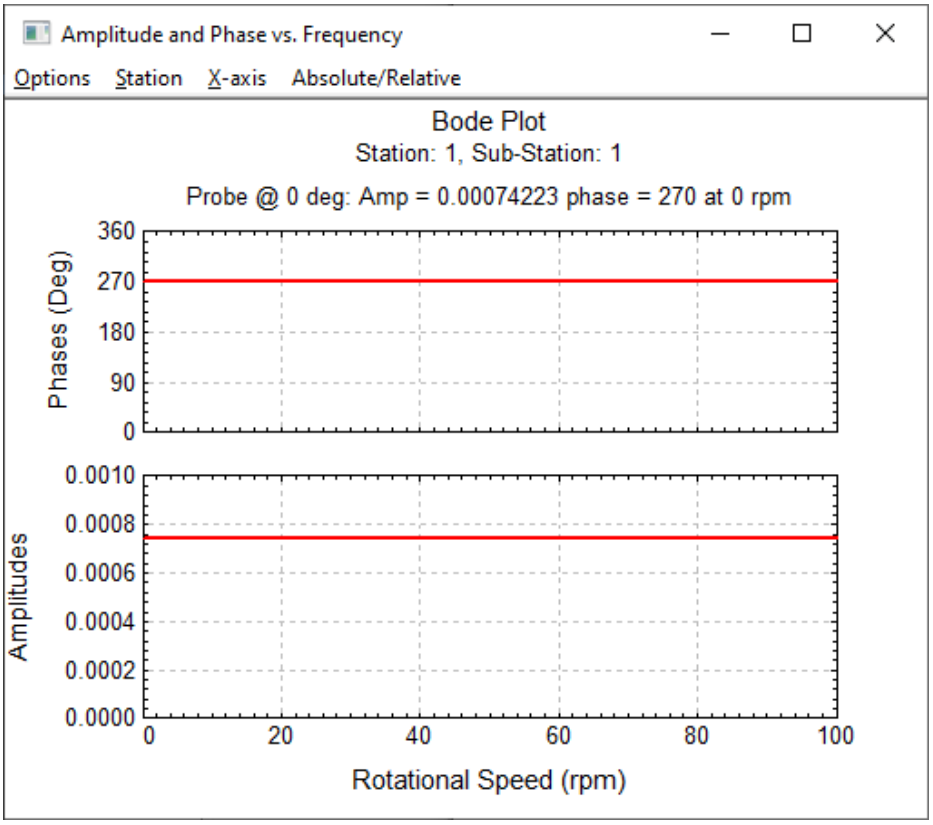


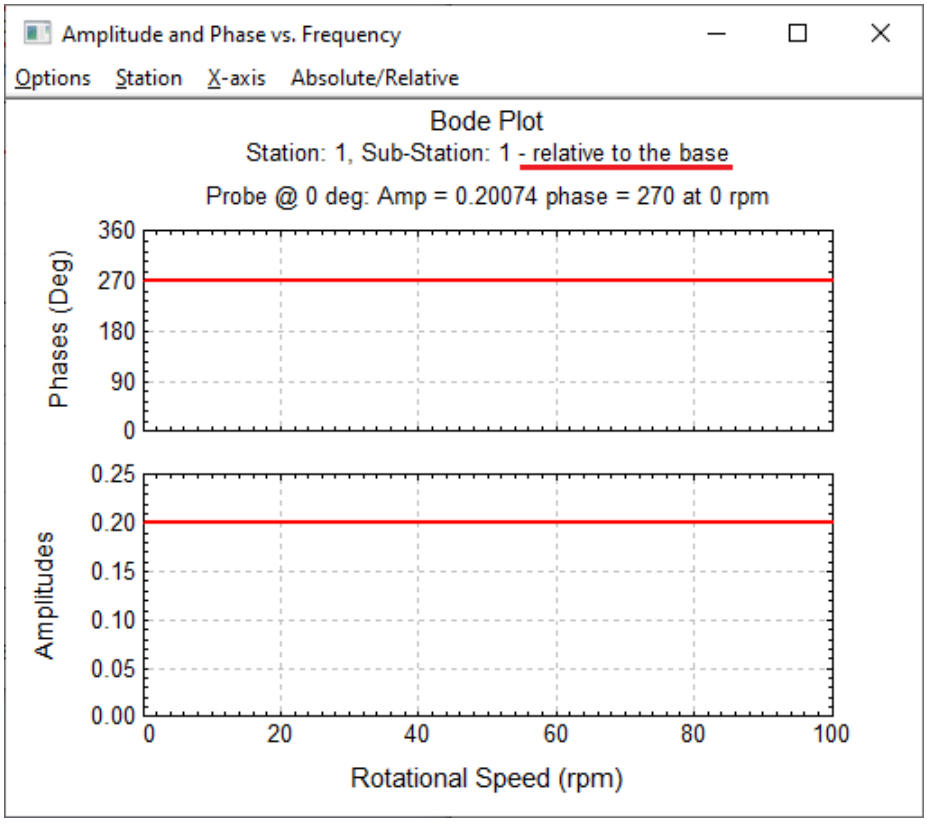
Fig. 9 – Input data for Example 1 Case 1 (BaseMotion_1a.rot)

The constraints are used to limit the system to be a single degree-of-freedom in the X direction only for the purposes of comparison and understanding the base motion.

The reason to run the analysis from 0 to 100 rpm with an increment of 10 rpm is primarily for the easy presentation of the results. For this simple system, all the system parameters are not dependent on the rotor speed, therefore the results are also independent of the speed.

The results, the absolute displacement and the relative displacement of the mass and the force transmitted to the base, from the Base Motion Analysis are shown below:


Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Bode Plot
Station: 1, Sub-Station: 1
Probe @ 0 deg: Amp = 0.00074223 phase = 270 at 0 rpm
Phases (Deg)
360
270
180
90
0
Amplitudes
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
0
20
40
60
80
100
Rotational Speed (rpm)


Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Bode Plot
Station: 1, Sub-Station: 1 - relative to the base
Probe @ 0 deg: Amp = 0.20074 phase = 270 at 0 rpm
Phases (Deg)
360
270
180
90
0
Amplitudes
0.25
0.20
0.15
0.10
0.05
0.00
0
20
40
60
80
100
Rotational Speed (rpm)

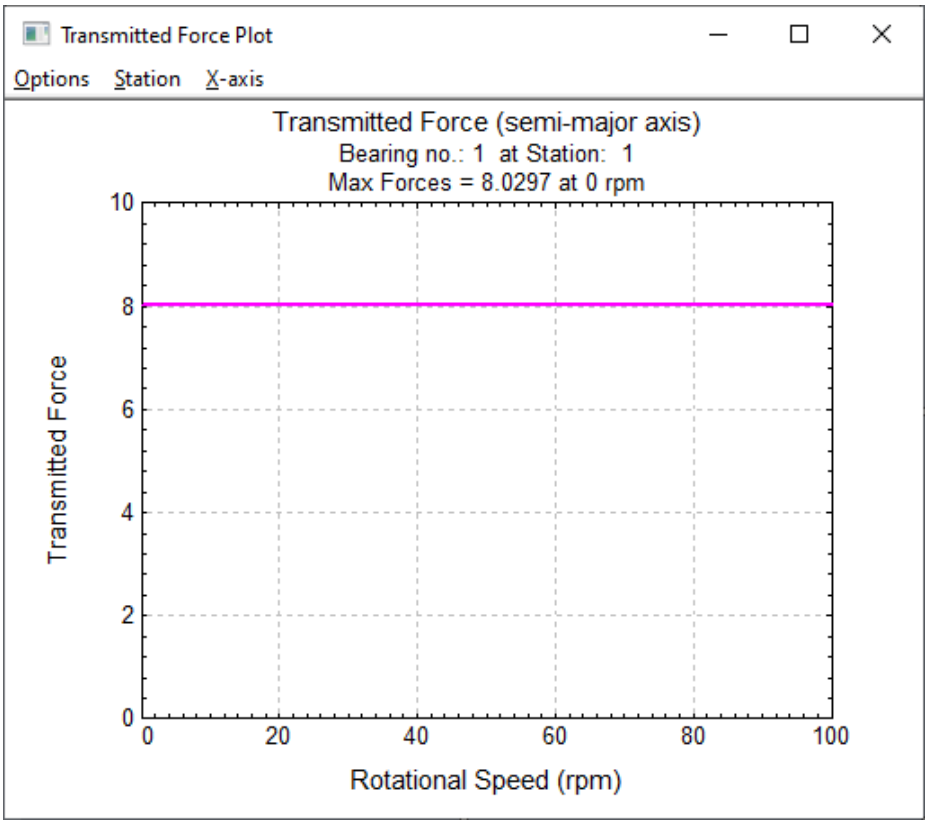


Fig. 10 – Outputs for Example 1 Case 1 (BaseMotion_1a.rot)

As said that the absolute displacement can be verified by using the Steady State Harmonic Excitation Analysis according to Eq. (8). The steady state harmonic excitation is a function of bearing coefficients (k and c) and base motion (z and ω_{exc}). However, the force transmitted to the base through the bearing cannot be verified using this analysis, since the relative displacement is not obtainable from the Steady State Harmonic Excitation Analysis without knowing the base motion.

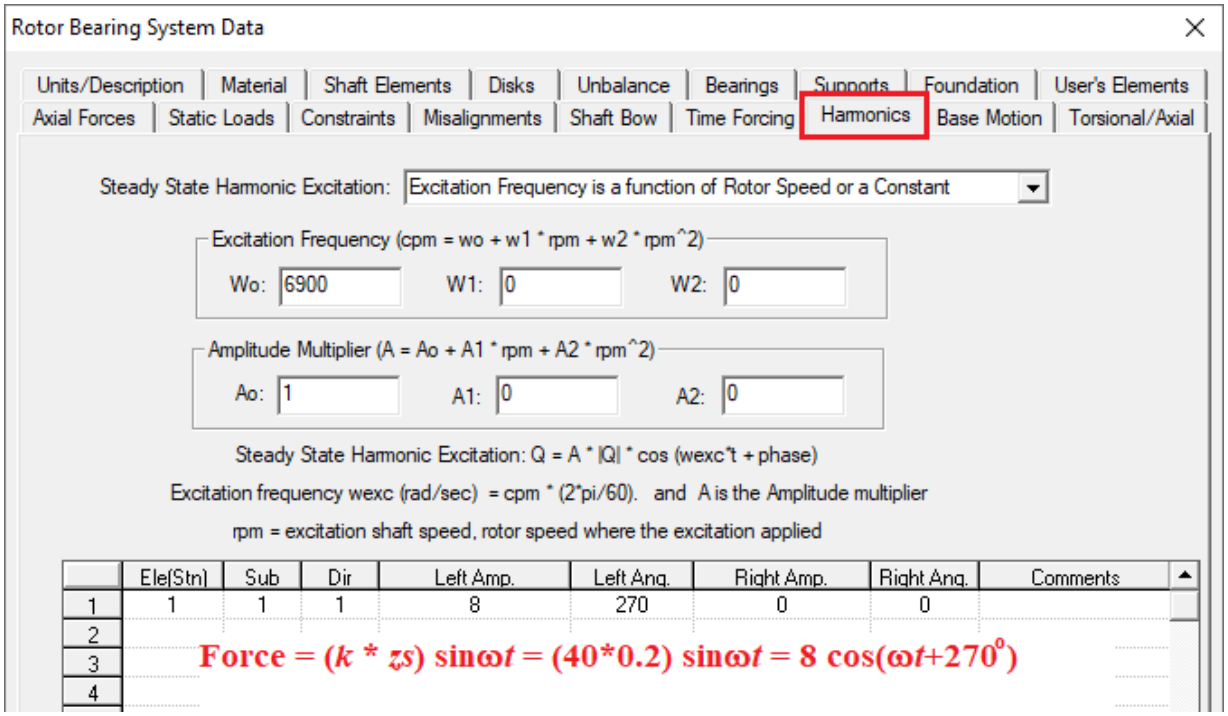
The bearing transmitted force from Base Motion is:

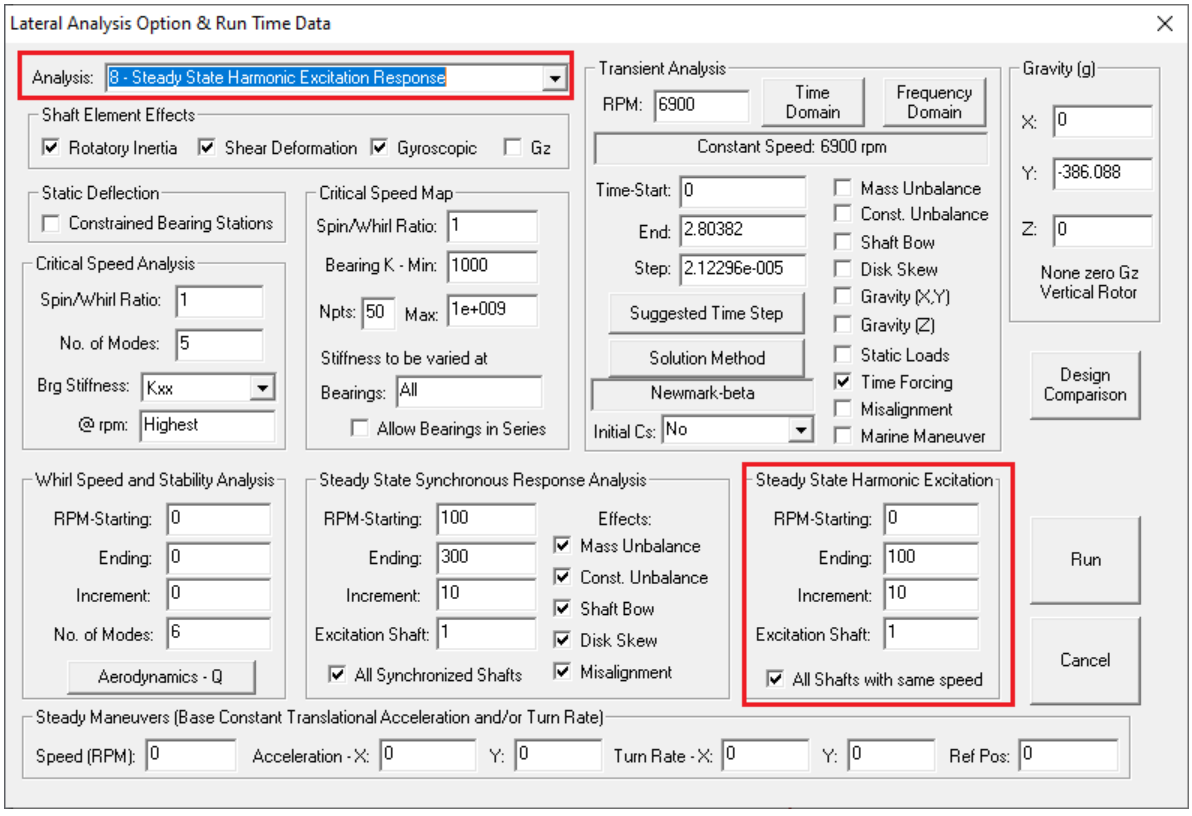
$$F = k\left(x - z\right) + c\left(\dot{x} - \dot{z}\right) \tag{11}$$

The bearing transmitted force from the Steady State Harmonic Excitation is:

$$F = kx + c\dot{x} \tag{12}$$

The input and output for the steady state harmonic analysis are show below. Note that the force expression uses a phase lead (ϕ) and the displacement uses a phase lag.


Rotor Bearing System Data
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Steady State Harmonic Excitation: Excitation Frequency is a function of Rotor Speed or a Constant
Excitation Frequency (cpm = wo + w1 * rpm + w2 * rpm^2)
Wo: 6900
W1: 0
W2: 0
Amplitude Multiplier (A = Ao + A1 * rpm + A2 * rpm^2)
Ao: 1
A1: 0
A2: 0
Steady State Harmonic Excitation: Q = A * |Q| * cos (wexc*t + phase)
Excitation frequency wexc (rad/sec) = cpm * (2*pi/60). and A is the Amplitude multiplier
rpm = excitation shaft speed, rotor speed where the excitation applied
Ele[Stn] Sub Dir Left Amp. Left Ang. Right Amp. Right Ang. Comments
1 1 1 1 8 270 0 0
2
3
4
Force = (k * zs) sinωt = (40*0.2) sinωt = 8 cos(ωt+270°)


Lateral Analysis Option & Run Time Data
Analysis: 8 - Steady State Harmonic Excitation Response
Shaft Element Effects
Rotatory Inertia
Shear Deformation
Gyroscopic
Gz
Static Deflection
Constrained Bearing Stations
Critical Speed Analysis
Spin/Whirl Ratio: 1
No. of Modes: 5
Brg Stiffness: Kxx
@ rpm: Highest
Critical Speed Map
Spin/Whirl Ratio: 1
Bearing K - Min: 1000
Npts: 50
Max: 1e+009
Stiffness to be varied at
Bearings: All
Allow Bearings in Series
Transient Analysis
RPM: 6900
Time Domain
Frequency Domain
Constant Speed: 6900 rpm
Time-Start: 0
End: 2.80382
Step: 2.12296e-005
Suggested Time Step
Solution Method
Newmark-beta
Initial Cs: No
Mass Unbalance
Const. Unbalance
Shaft Bow
Disk Skew
Gravity (X,Y)
Gravity (Z)
Static Loads
Time Forcing
Misalignment
Marine Maneuver
Gravity (g)
X: 0
Y: -386.088
Z: 0
None zero Gz Vertical Rotor
Design Comparison
Whirl Speed and Stability Analysis
RPM-Starting: 0
Ending: 0
Increment: 0
No. of Modes: 6
Aerodynamics - Q
Steady State Synchronous Response Analysis
RPM-Starting: 100
Ending: 300
Increment: 10
Excitation Shaft: 1
All Synchronized Shafts
Effects:
Mass Unbalance
Const. Unbalance
Shaft Bow
Disk Skew
Misalignment
Steady State Harmonic Excitation
RPM-Starting: 0
Ending: 100
Increment: 10
Excitation Shaft: 1
All Shafts with same speed
Run
Cancel
Steady Maneuvers (Base Constant Translational Acceleration and/or Turn Rate)
Speed (RPM): 0
Acceleration - X: 0
Y: 0
Turn Rate - X: 0
Y: 0
Ref Pos: 0

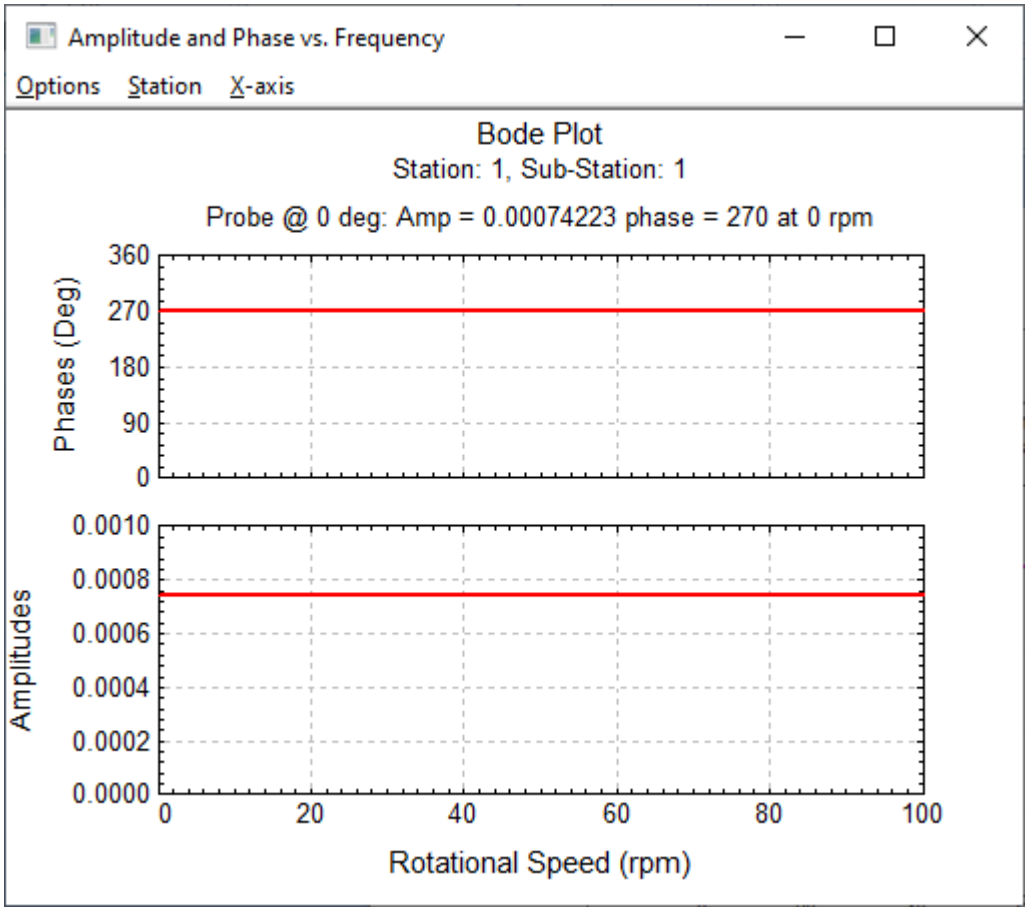


Fig. 10 – Verification of Base Motion using the Steady State Harmonic Excitation Analysis

<u>Case 2</u>: BaseMotion_1b.rot

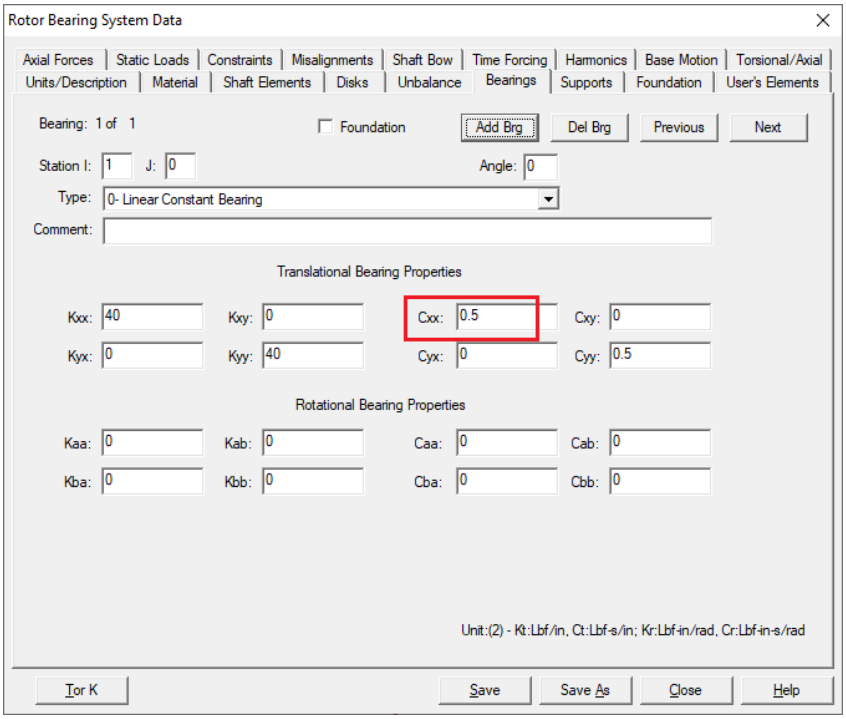
Since this simple system has a constant and speed independent natural frequency, let us consider a wide range of excitation frequency at a constant rotor speed to plot the response versus the frequency ratio. To limit the response amplitude at the resonance, frequency ratio =1, let us add a damping coefficients of c = 0.5 in the bearing. Then we have:

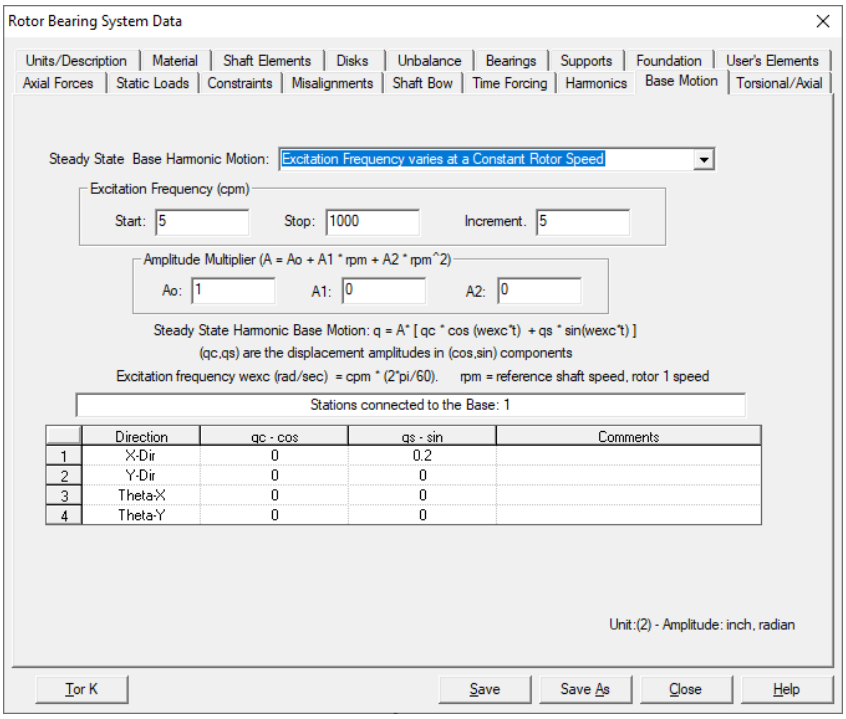
$$\text{undamped natural frequency} = \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{8/386.088}} = 43.94 \text{ rad/sec} = 419.57 \text{ cpm}$$

$$\text{damping factor} = \zeta = \frac{c}{2m\omega_n} = \frac{0.5}{2\times\frac{8}{386.088}\times 43.94} = 0.275$$

$$\text{damped natural frequency} = \omega_d = \omega_n\sqrt{1-\zeta^2} = 42.25 \text{ rad/sec} = 403 \text{ cpm}$$

The relevant inputs are shown in Fig. 11 below:





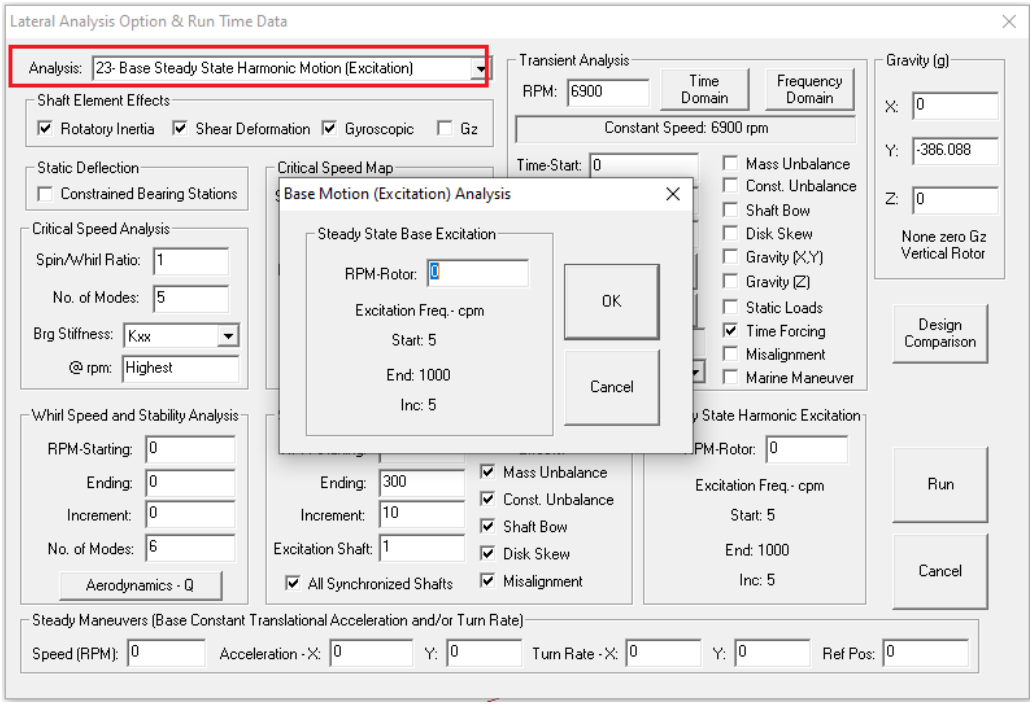
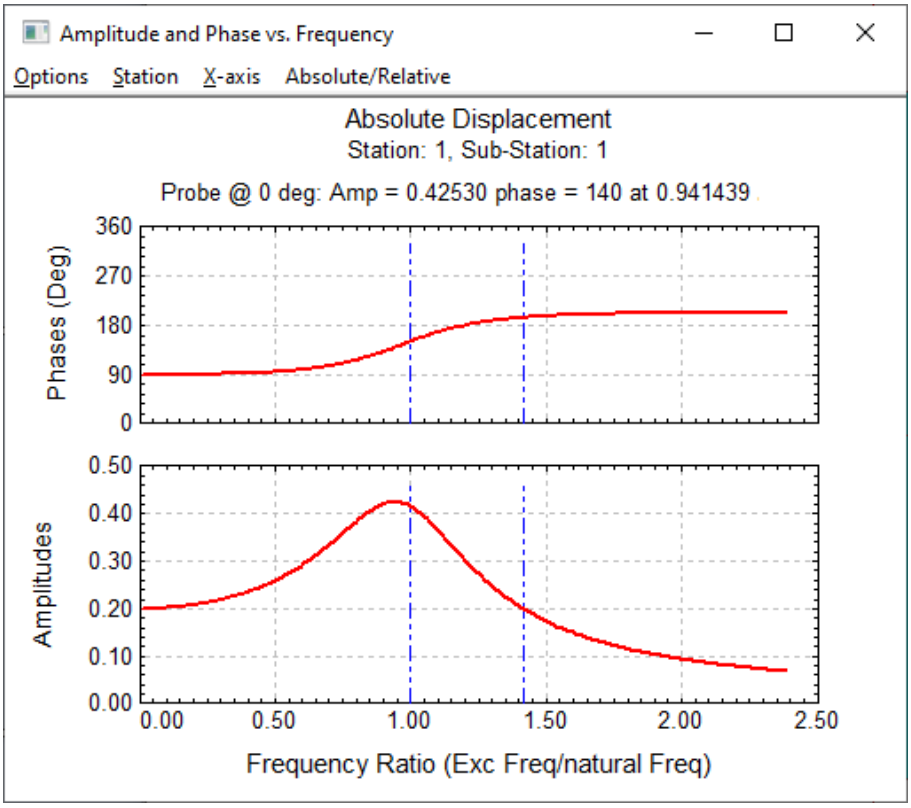


Fig. 11 – Example 1 Case 2: Excitation frequency from 5 to 1000 cpm

The absolute and relative displacements of the mass and the force transmitted to the base versus the frequency ratio are shown in Fig. 12 below:



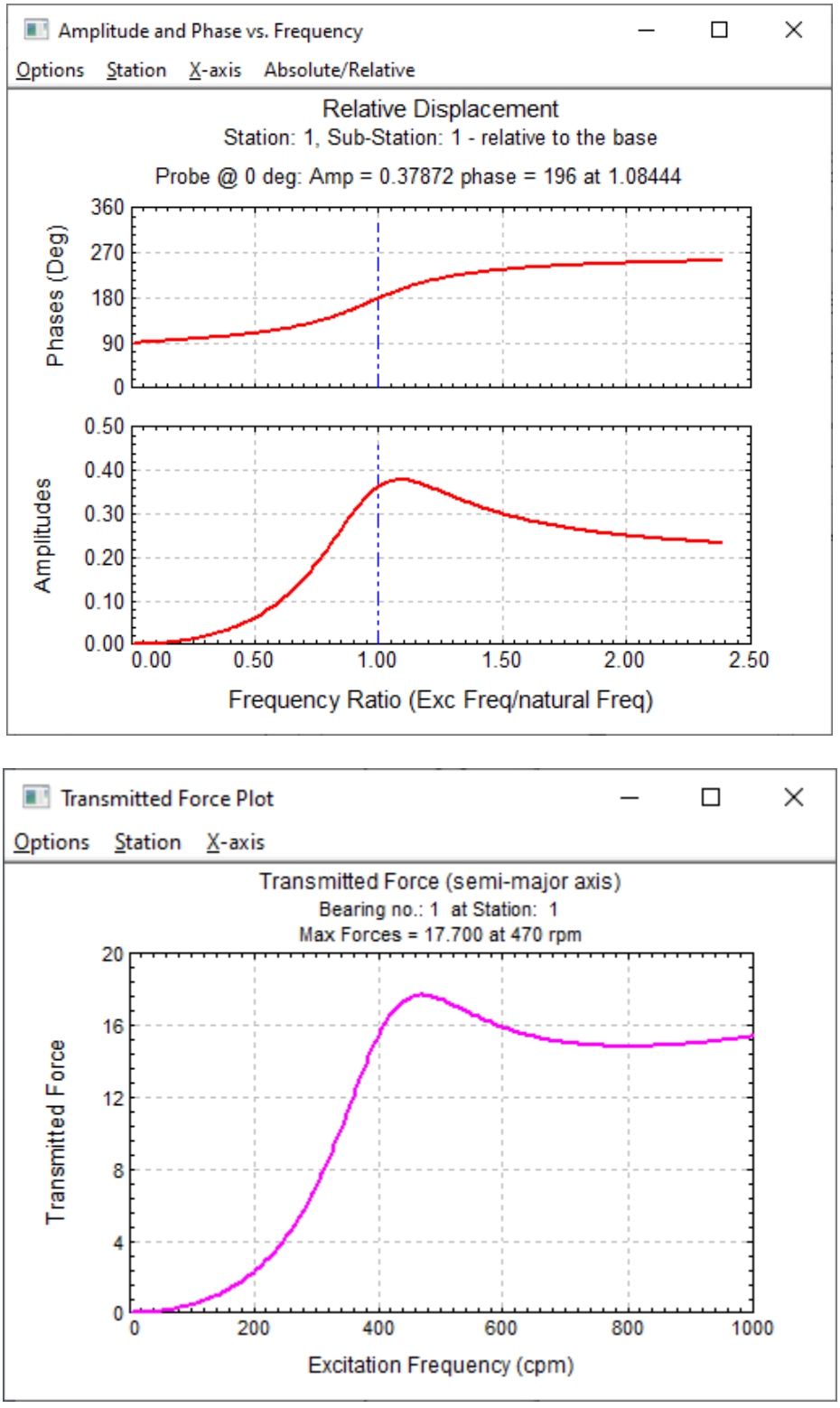
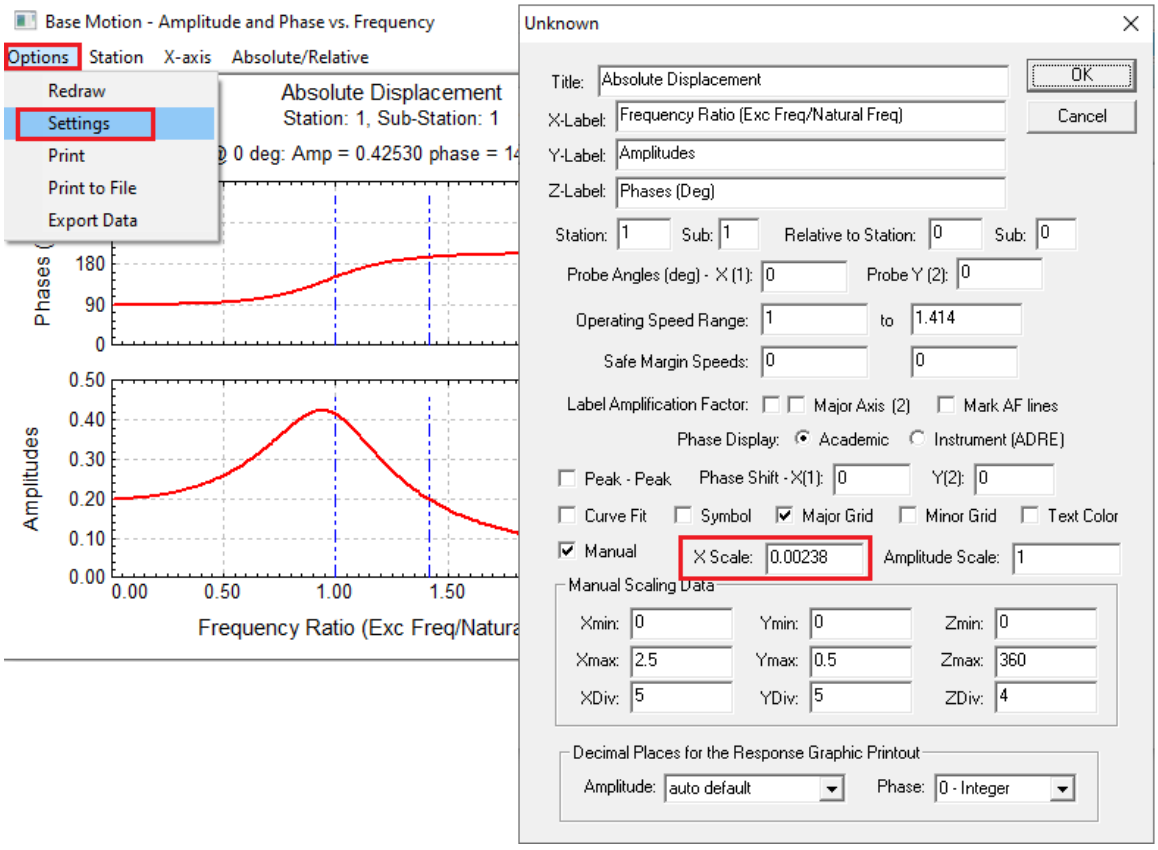


Fig. 12 – Base Motion Results for Example 1 Case 2

To change the graphic scales and headings, go to Options – Settings and macke necessary changes. To scale the X-axis from Excitation Frequency to Frequency Ratio (Exec Freq/natural Freq), multiplying a scale factor (1/Natural Freq = 1/419.57=0.00238) as shown below.



As expected, the absolute displacement starts with the base motion at γ =0 and reaches the maximum absolute displacement before the resonance γ =1, and the absolute displacement equals to the base motion of 0.2 inches at γ =0, and $\sqrt{2}$. The absolute displacement decreases as the frequency ratio increases after the resonance. The relative displacement starts from zero and reaches the maximum relative displacement after the resonance and approaches to the base motion when the frequency ratio is very high. At very low frequency ratio, i.e., the excitation frequency is much less than the system natural frequency, the mass vibrates with the base and there is little relative motion between the mass and the base. At very high frequency ratio, the mass is nearly stationary and the relative motion is primary the base motion.

Again, the maximum absolute displacement, 0.4253 inches at 395 rpm can be verified by using the steady state harmonic excitation analysis. The excitation frequency is 395 cpm (41.36 rad/sec) and the excitation force, according to Eq. (8), is:

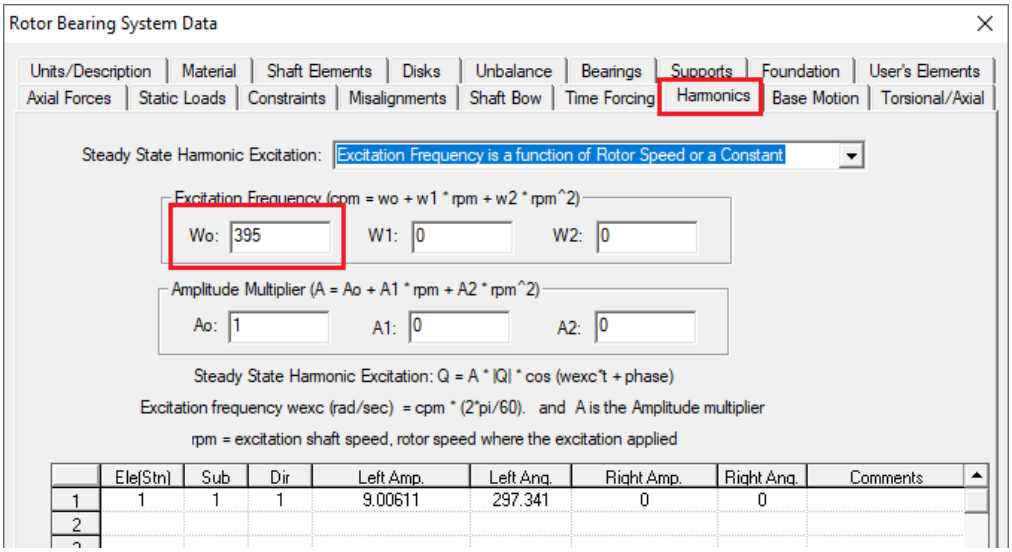
$$kz + c\dot{z} = k\left(z_c \cos\omega_{exc}t + z_s \sin\omega_{exc}t\right) + c\omega_{exc}\left(z_s \cos\omega_{exc}t - z_c \sin\omega_{exc}t\right)$$

$$= 40\left(0.2\sin\omega_{exc}t\right) + 0.5\times\frac{395\times 2\pi}{60}\left(0.2\cos\omega_{exc}t\right)$$

$$= 8\sin\omega_{exc}t + 4.13643\cos\omega_{exc}t = 9.00611\ \cos\left(\omega_{exc}t - 62.659^o\right)$$

$$= 9.00611\ \cos\left(\omega_{exc}t + 297.341^o\right)$$

The input and output are shown below:



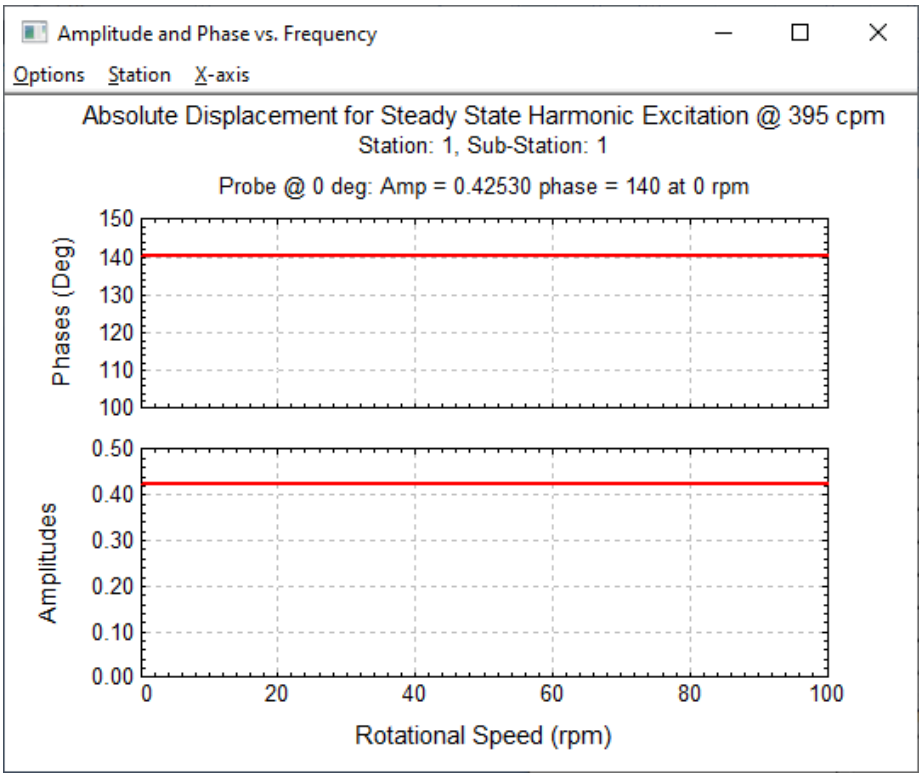
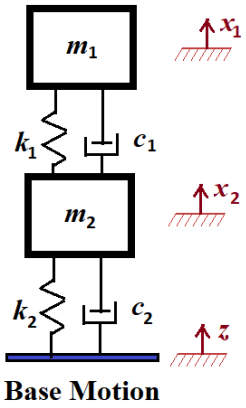


Fig. 13 – Steady State Harmonic Excitation at 395 cpm for Example 1 case 2

Example 2 - 2 Degrees-of-Freedom

The second example is a 2 DOF system, as shown below. For more details, see "Structural Dynamics" by Roy R. Craig, Jr., page 240.



m_1=8 lb, k_1 = 40 lb/in, c_1 = 0.2 lb-sec/in
m_2=15 lb, k_2 = 100 lb/in, c_2 = 0.1 lb-sec/in
base motion: z_c =0, z_s = 0.2, ω_{exc} from 5 cpm to 1000 cpm

Fig. 14 – Two Degrees-of-Freedom System

The equation of motion:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} c_1 & -c_1 \\ -c_1 & (c_1+c_2) \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}+\begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2) \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} 0 \\ k_2 z+c_2\dot{z} \end{Bmatrix} \tag{13}$$

Define the relative displacements with respect to the base motion z:

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}=\begin{Bmatrix} x_1-z \\ x_2-z \end{Bmatrix} \quad \Rightarrow \quad \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}+\begin{Bmatrix} z \\ z \end{Bmatrix} \tag{14}$$

Substitution of Eq. (14) into Eq. (13), the equation of motion in the relative displacement form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix}+\begin{bmatrix} c_1 & -c_1 \\ -c_1 & (c_1+c_2) \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix}+\begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1+k_2) \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}=-\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{z} \\ \ddot{z} \end{Bmatrix} \tag{15}$$

The response can be solved in either coordinate system (absolute or relative displacements). For a single base model, it is convenient to solve the equation of motion in the relative displacement, Eq. (15). However, for a multiple bases model, will be presented later, the absolute displacement, Eq. (13) will be used.

Again, this is also a simple model and the system parameters are independent from the rotor speed. The system natural frequencies are constant and not varied with the rotor speed.

The system natural frequencies and damping factors can be obtained from the whirl speed analysis:

```
******************** Whirl Speed and Stability Analysis ********************
*************************** Precessional Modes ***************************
          ************ Frequency *************     Damping      Log.    Damping
    Mode     rpm           R/S           Hz       Coefficient  Decrement Factor

     1      321.291       33.6456       5.3549      -1.556       .291     0.046
     2      628.304       65.7958       10.472      -7.131       .681     0.108
 ************************************************************************
```

Undamped Natural Frequencies:

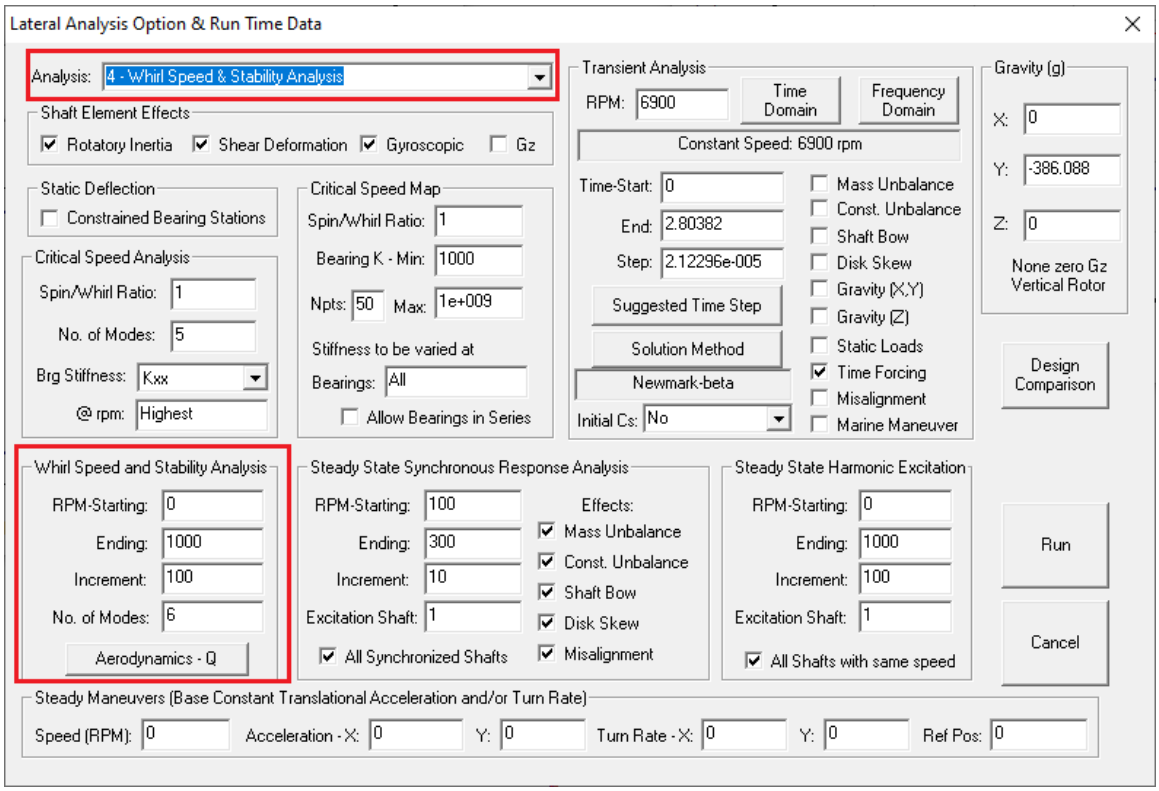
$\omega_{n1} = 33.58$ rad/sec = 320.67 cpm

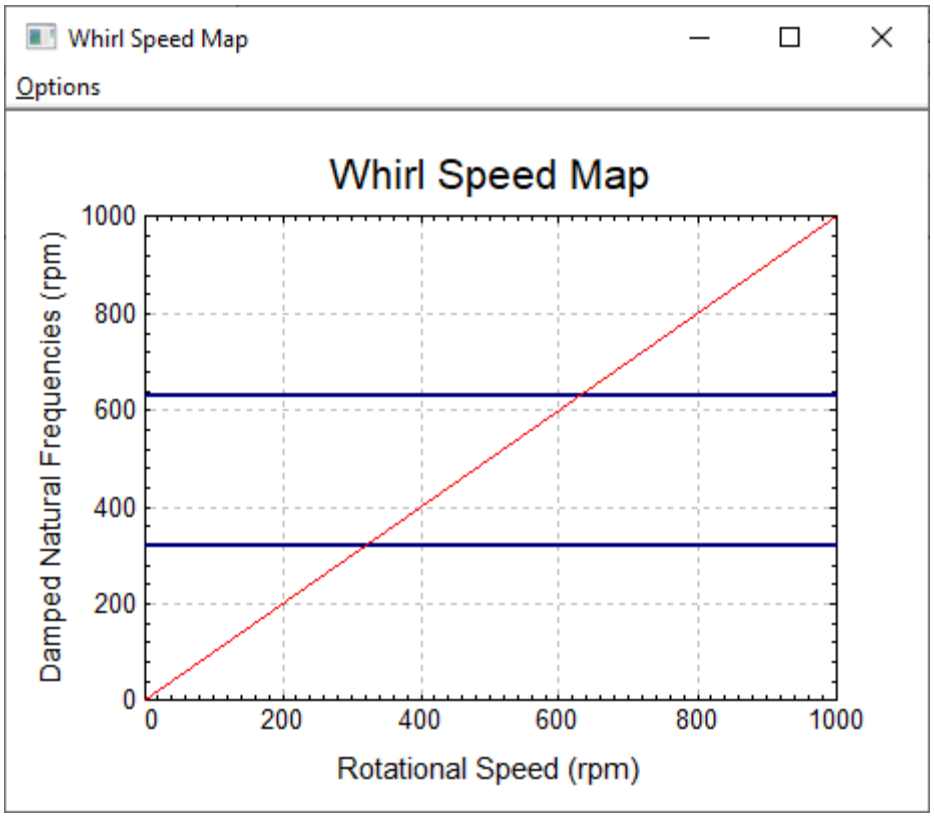
$\omega_{n2} = 66.38$ rad/sec = 633.8 cpm

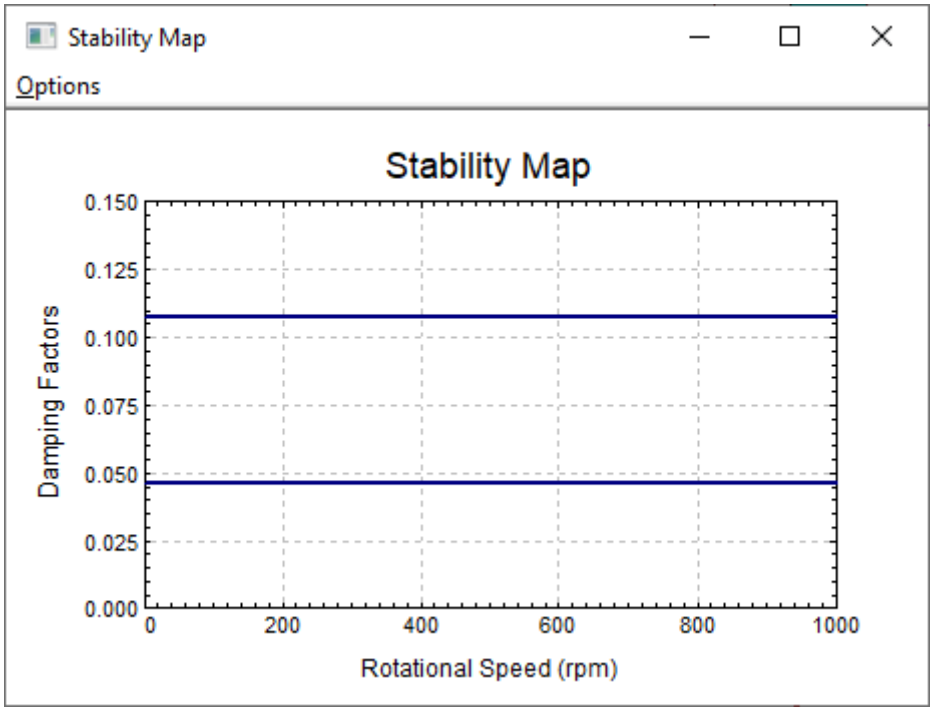
Damped Natural Frequencies and Damping Factors:

$\omega_{n1} = 33.65$ rad/sec = 321.29 cpm $\qquad \zeta_1 = 0.046$

$\omega_{n2} = 65.80$ rad/sec = 628.30 cpm $\qquad \zeta_2 = 0.108$

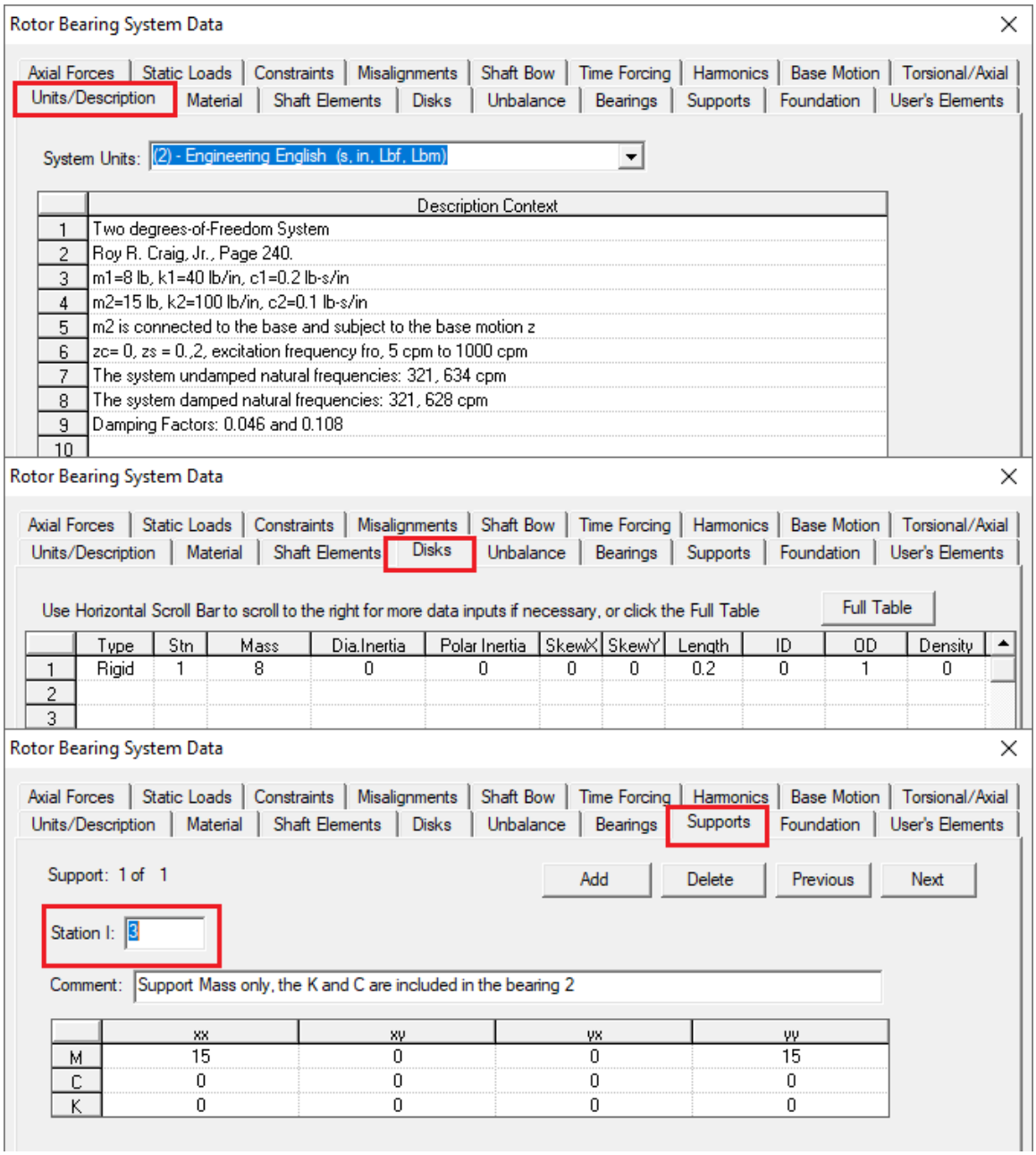


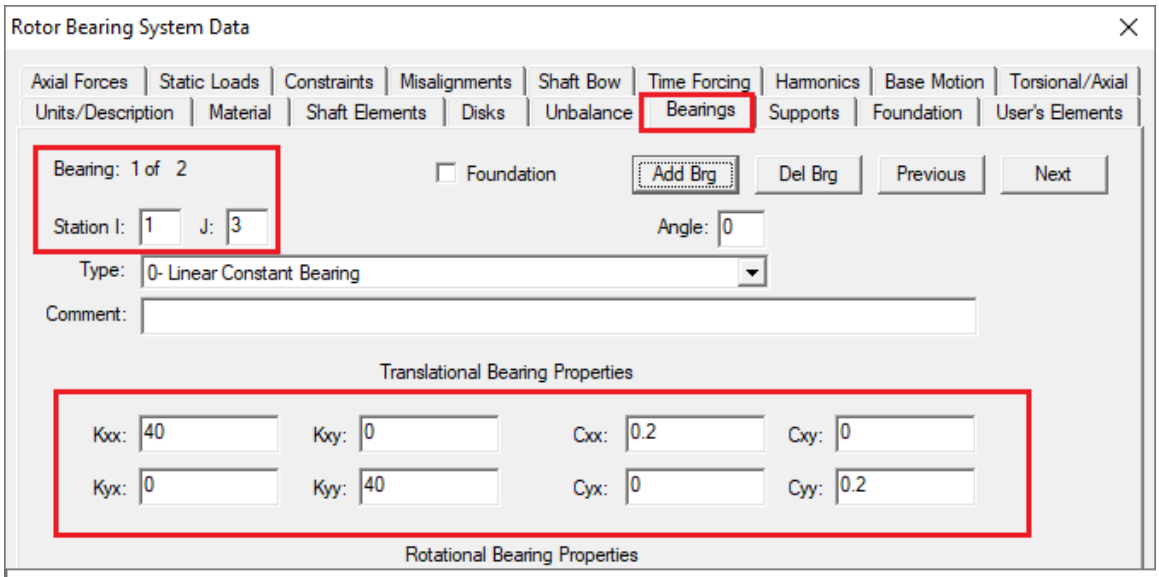

Whirl Speed Map
Options
Whirl Speed Map
Damped Natural Frequencies (rpm)
1000
800
600
400
200
0
0
200
400
600
800
1000
Rotational Speed (rpm)

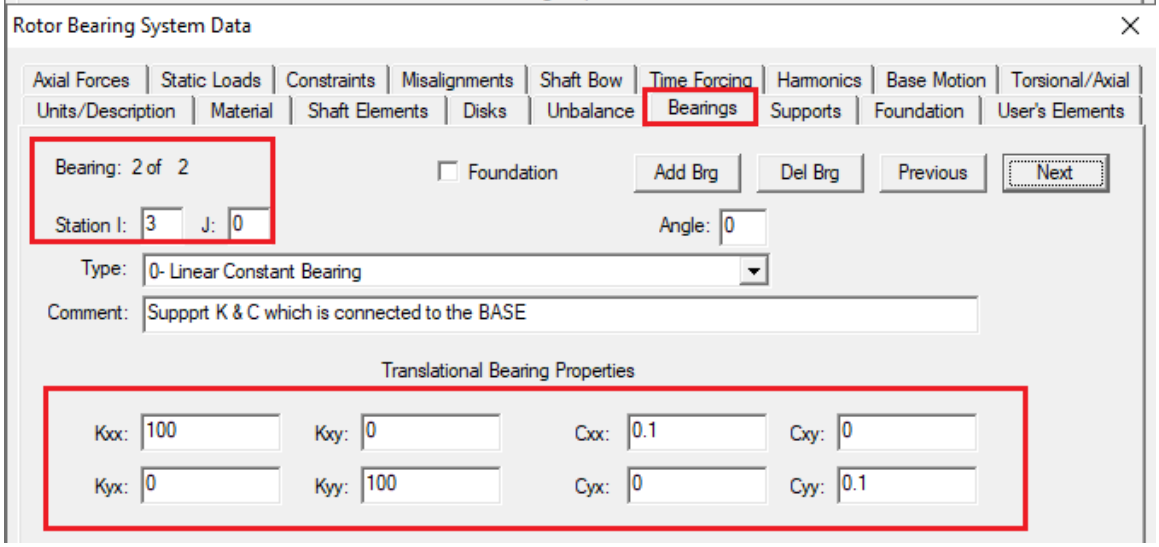

Stability Map
Options
Stability Map
Damping Factors
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
200
400
600
800
1000
Rotational Speed (rpm)

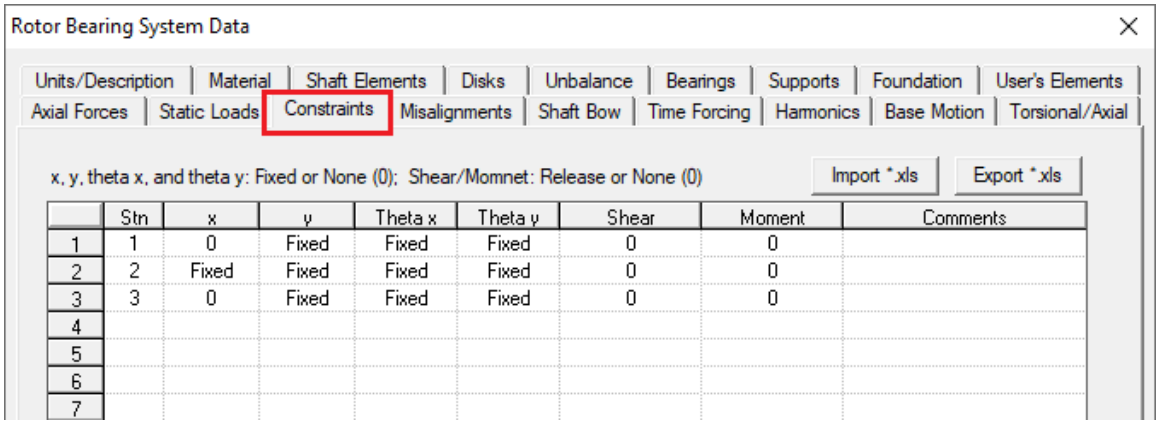
Case 1: BaseMotion_2a.rot

The relevant inputs are shown below in Fig. 15. Note that in this model, the support stiffness and damping are entered as a bearing.




Rotor Bearing System Data
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Bearing: 1 of 2
Foundation
Add Brg
Del Brg
Previous
Next
Station I: 1 J: 3
Angle: 0
Type: 0- Linear Constant Bearing
Comment:
Translational Bearing Properties
Kxx: 40
Kxy: 0
Cxx: 0.2
Cxy: 0
Kyx: 0
Kyy: 40
Cyx: 0
Cyy: 0.2
Rotational Bearing Properties


Rotor Bearing System Data
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Bearing: 2 of 2
Foundation
Add Brg
Del Brg
Previous
Next
Station I: 3 J: 0
Angle: 0
Type: 0- Linear Constant Bearing
Comment: Suppprt K & C which is connected to the BASE
Translational Bearing Properties
Kxx: 100
Kxy: 0
Cxx: 0.1
Cxy: 0
Kyx: 0
Kyy: 100
Cyx: 0
Cyy: 0.1


Rotor Bearing System Data
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
x, y, theta x, and theta y: Fixed or None (0); Shear/Momnet: Release or None (0)
Import *.xls
Export *.xls
Stn x y Theta x Theta y Shear Moment Comments
1 1 0 Fixed Fixed Fixed 0 0
2 2 Fixed Fixed Fixed Fixed 0 0
3 3 0 Fixed Fixed Fixed 0 0
4
5
6
7

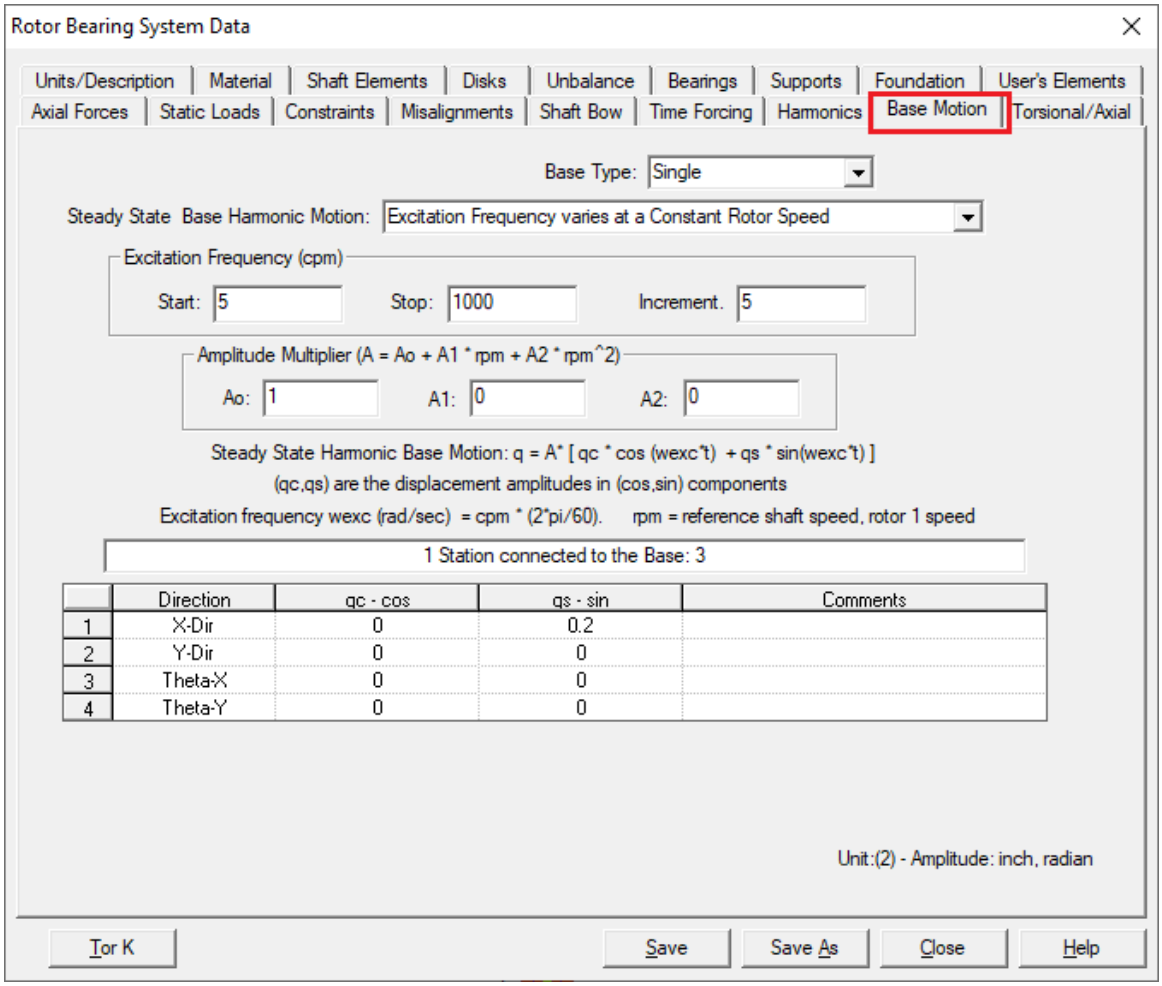
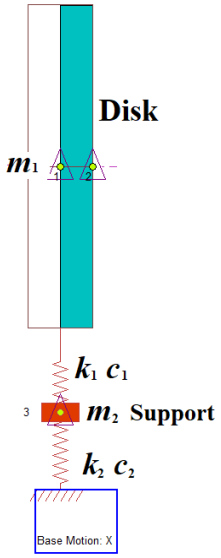


Fig. 15 – Inputs for Example 2



Mass m_1 absolute and relative displacements (station 1)

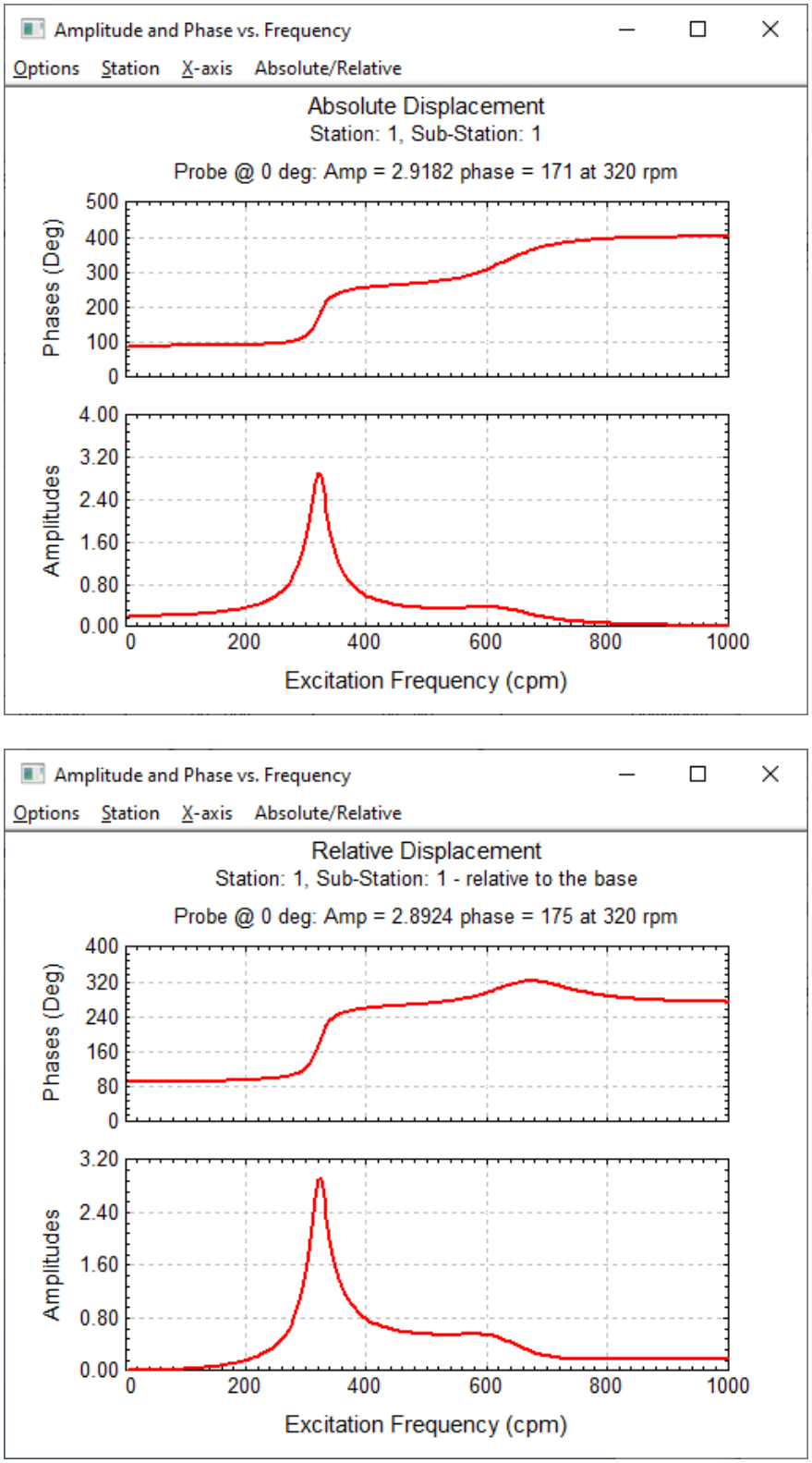


Fig. 16 – Displacements for Mass m_1

Mass m_2 absolute and relative displacements (station 3)

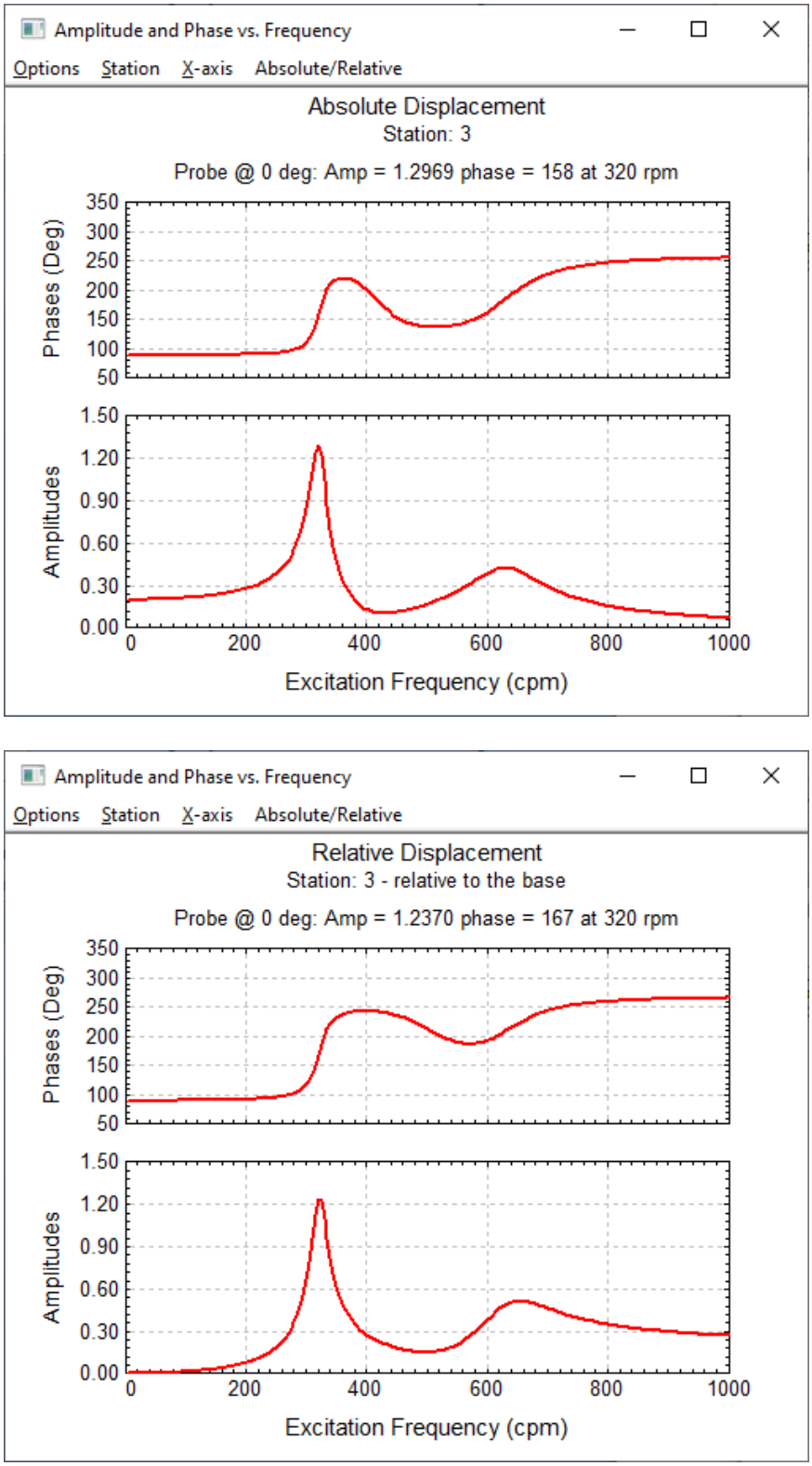


Fig. 17 – Displacements for Mass m_2

The transmitted forces for bearing 1 and bearing 2 are shown below. The force transmitted through bearing 2 is acting on the base.

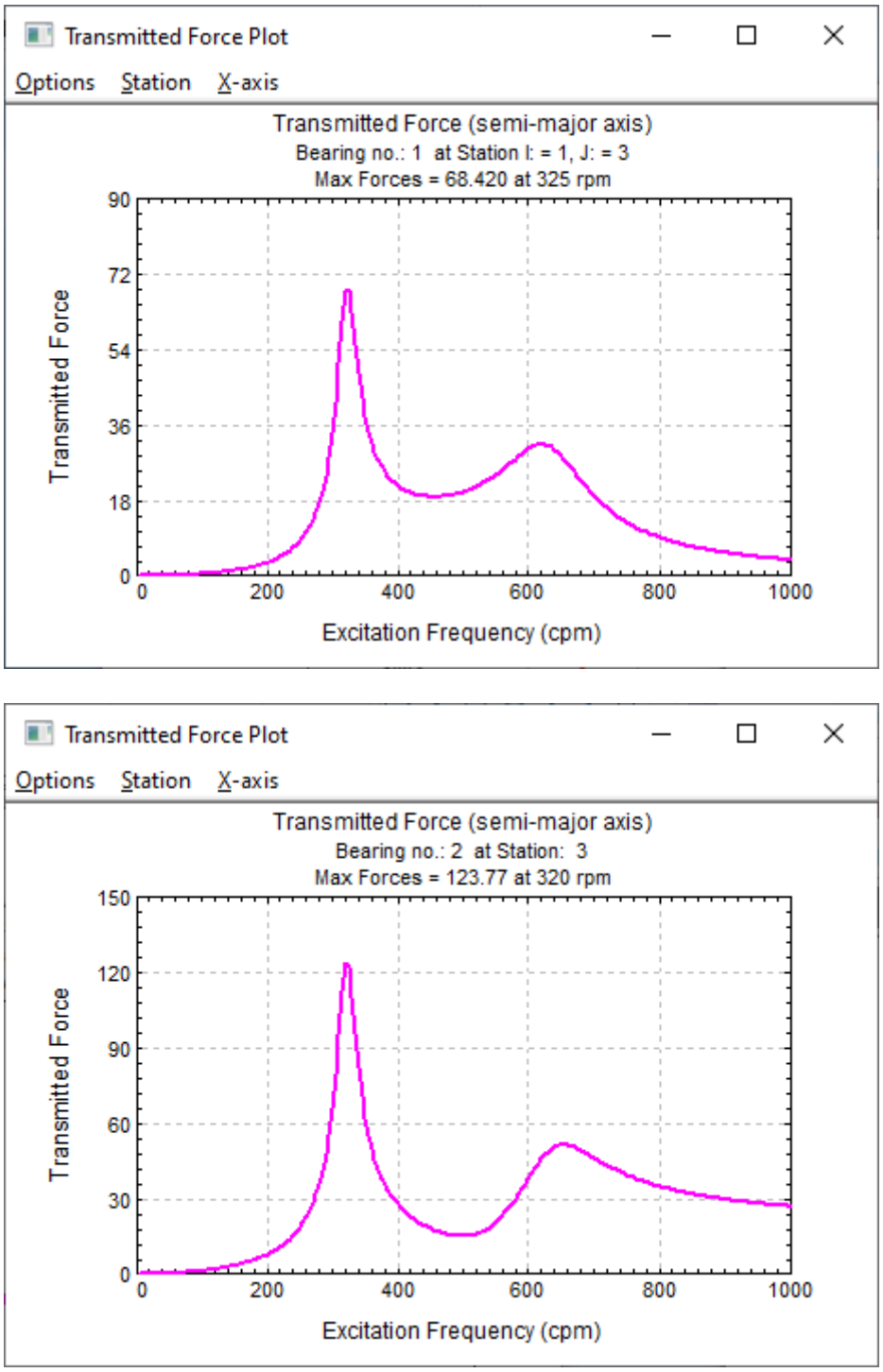


Fig. 18 – Forces Transmitted through Bearings

Again, the absolute displacements can be verified by using the steady state harmonic excitation analysis. The excitation acting on the mass m_2 (station 3) is:

$$kz + c\dot{z} = k\left(z_c \cos\omega_{exc}t + z_s \sin\omega_{exc}t\right) + c\omega_{exc}\left(z_s \cos\omega_{exc}t - z_c \sin\omega_{exc}t\right)$$
$$= \left(kz_c + c\omega_{exc}z_s\right)\cos\omega_{exc}t + \left(kz_s - c\omega_{exc}z_c\right)\sin\omega_{exc}t$$

For the excitation frequency of 320 cpm, the harmonic excitation is:

$$\left(0.1\times\frac{320\times 2\pi}{60}\times 0.2\right)\cos\omega_{exc}t + \left(100\times 0.2\right)\sin\omega_{exc}t$$

The harmonic excitation input:

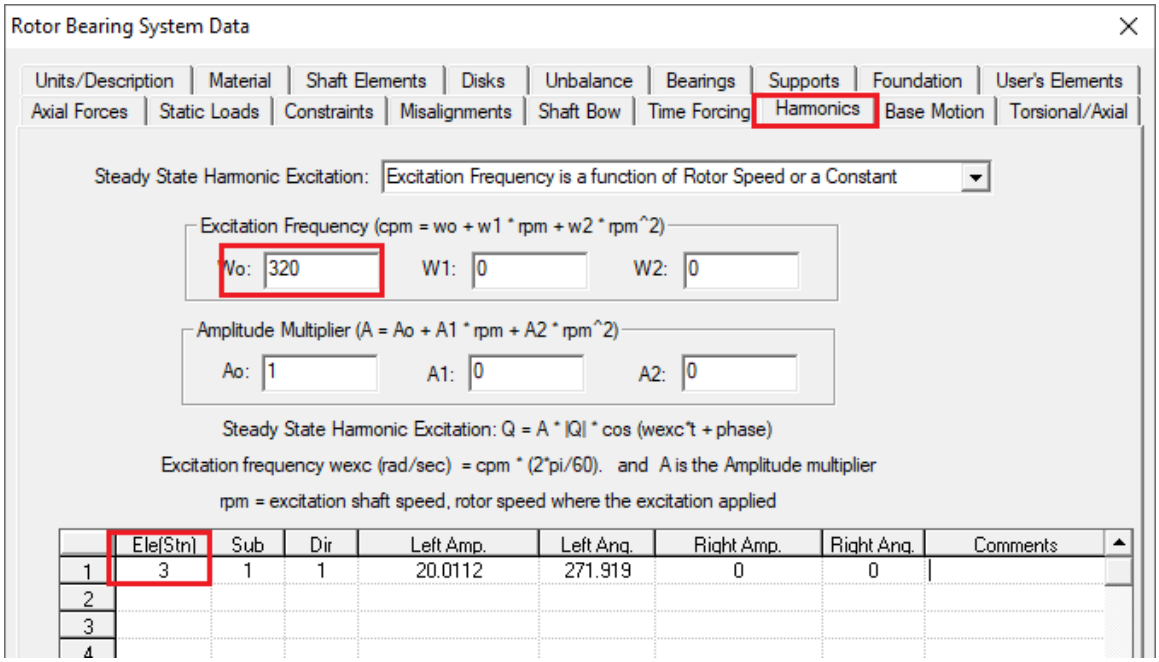


Fig. 19 – Harmonic Excitation at m_2 (station 3)

The results for the excitation frequency at 320 cpm can be verified. In order to shown the plots, we ran the steady state harmonic analysis for a speed range of 0-1000 rpm. The displacement results are:

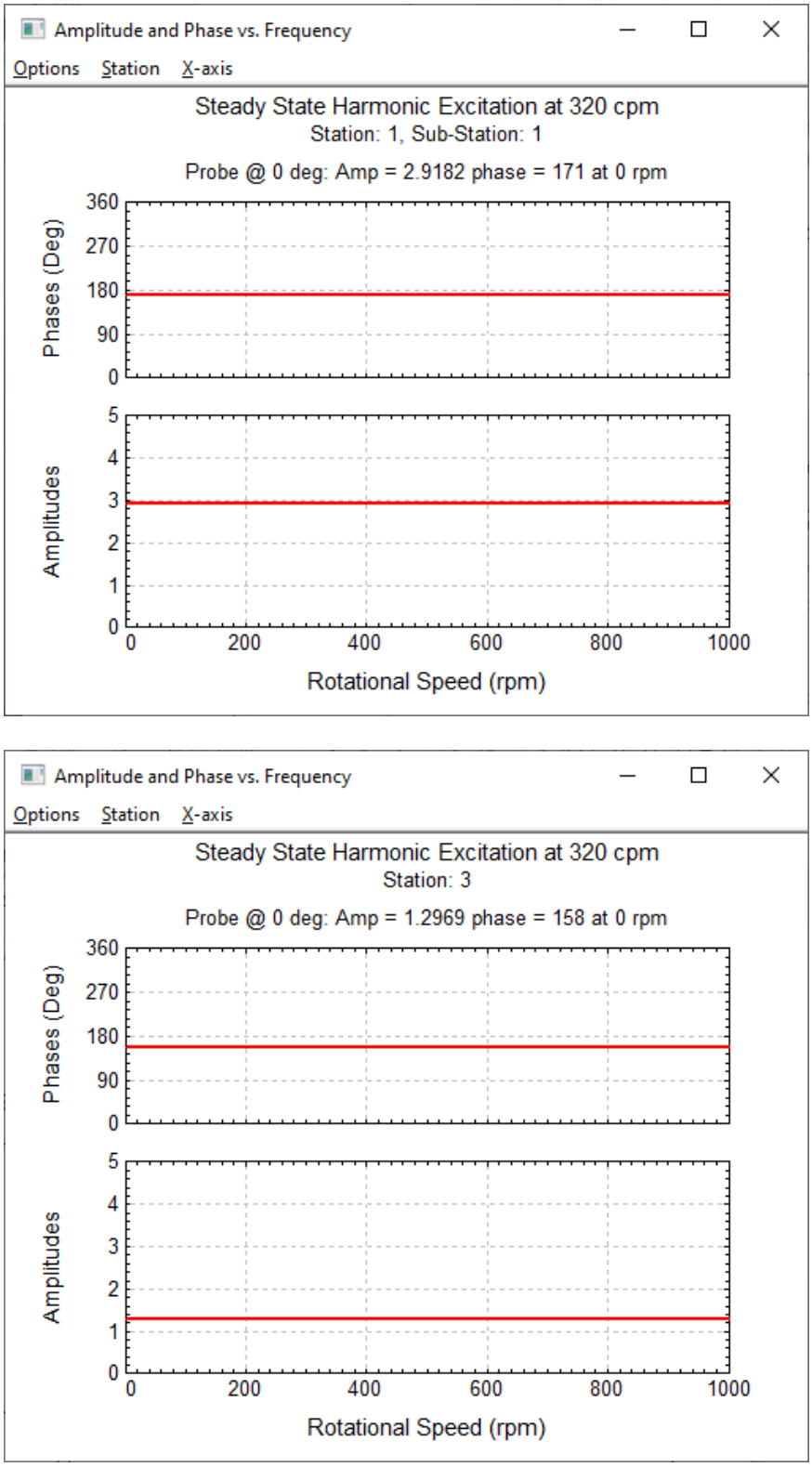


Fig. 20 – Responses for the Harmonic Excitation

Case 2: BaseMotion_2b.rot

In this model, we can also move the bearing 2 data into the support tab, as shown in Fig. 21. The results are identical to the previous model and not repeated here.

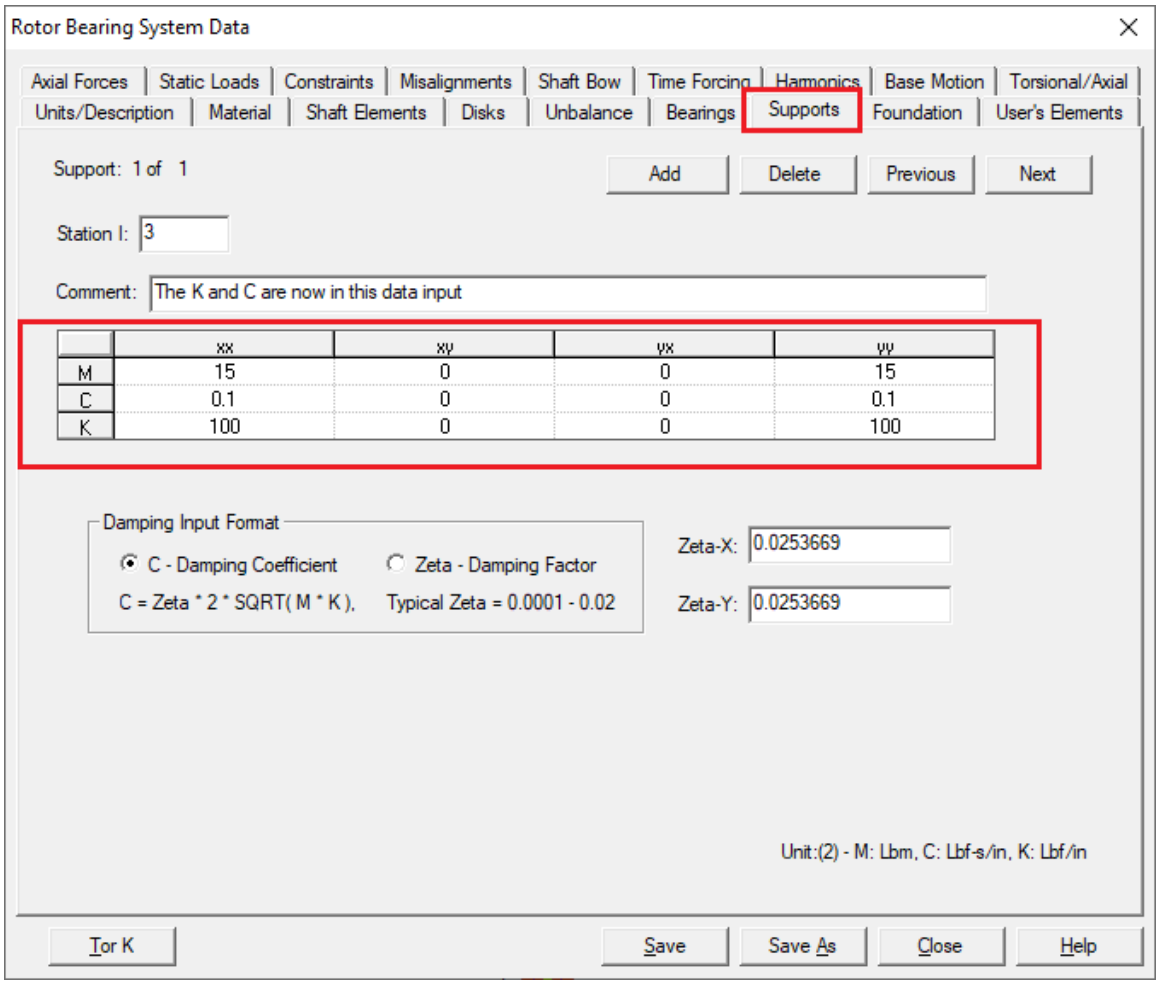


Fig. 21 – Alternative inputs for Bearing 2

Example 3 – Multiple Degrees-of-Freedom

A multiple DOF system, as shown in Fig. 22, is used in this example. At the operating speed of 3,000 rpm, the first five natural frequencies are: 4,312 (backward), 4,358 (forward), 7,342 (backward), 7,619 (forward), and 100,684 (backward) rpm and their associated whirling modes are shown in Fig. 23.

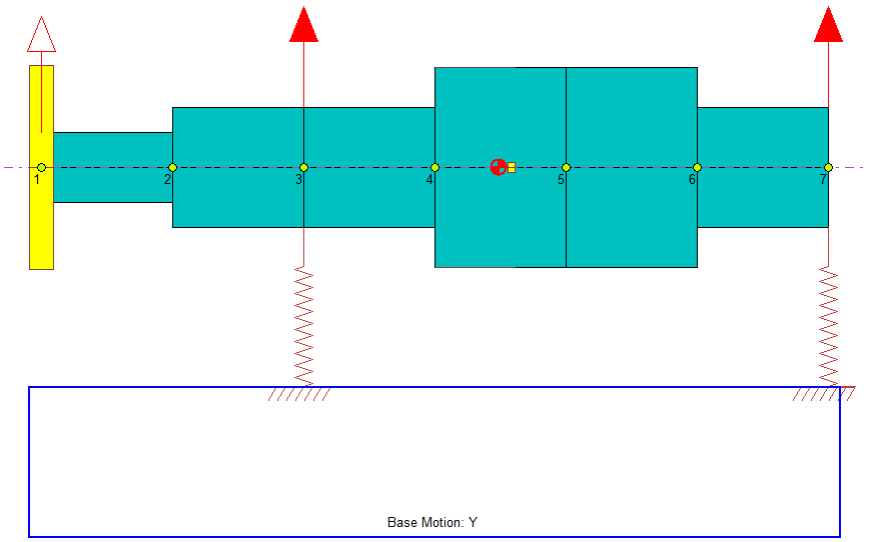
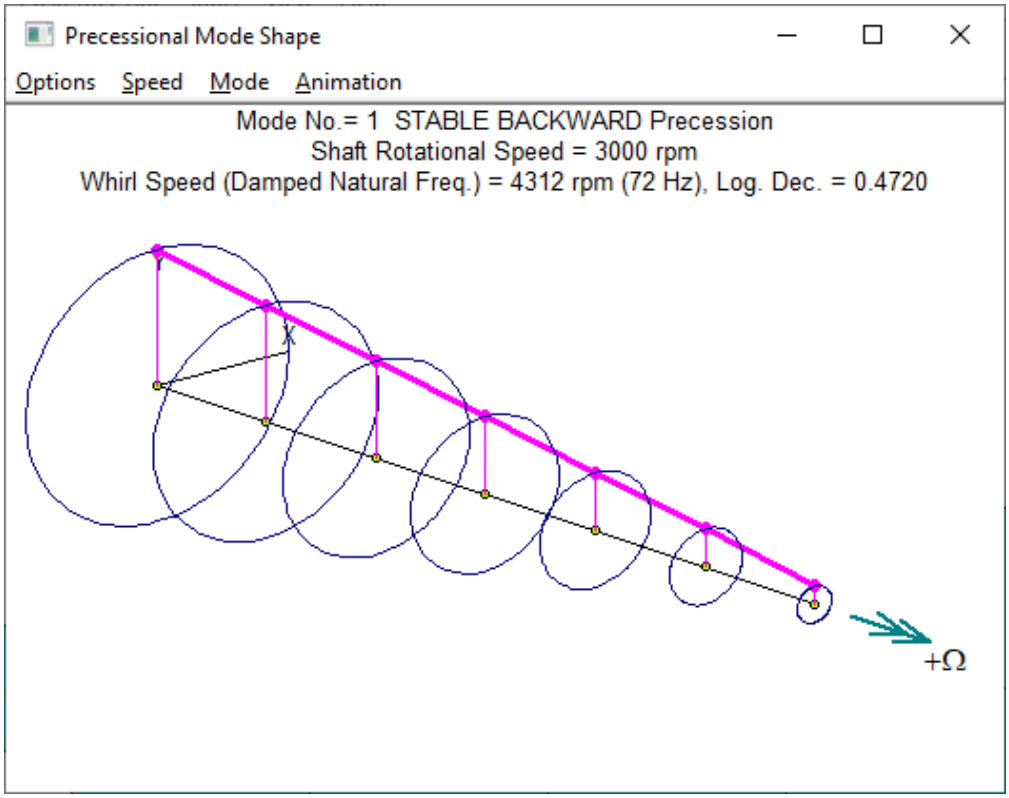
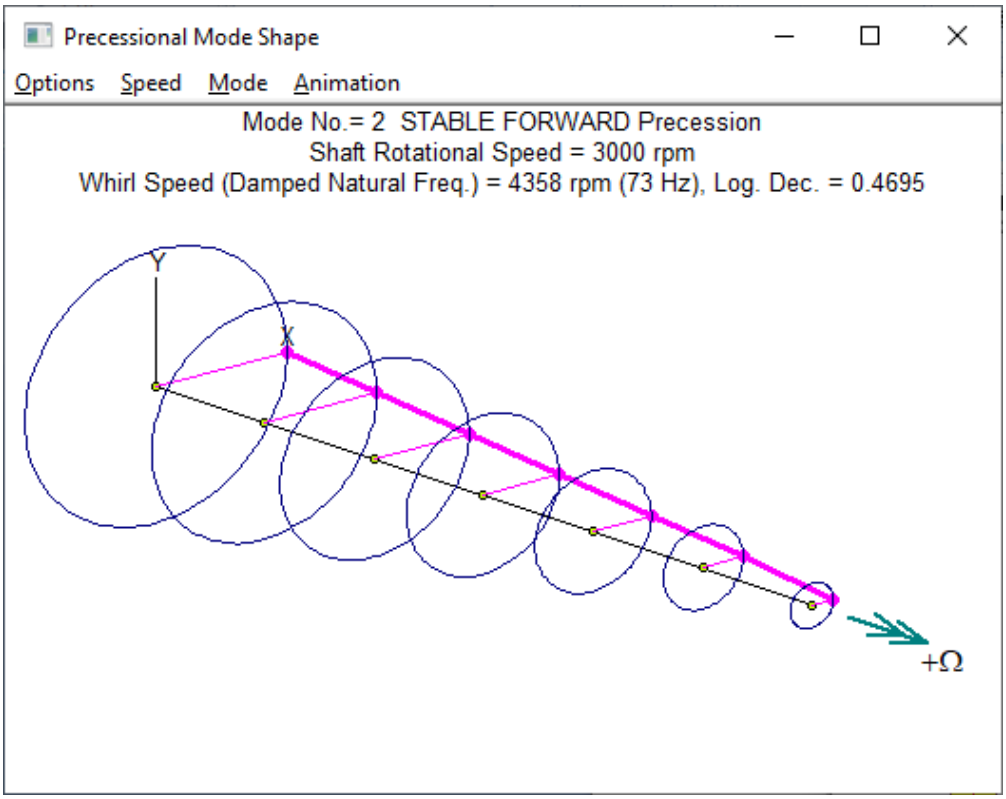
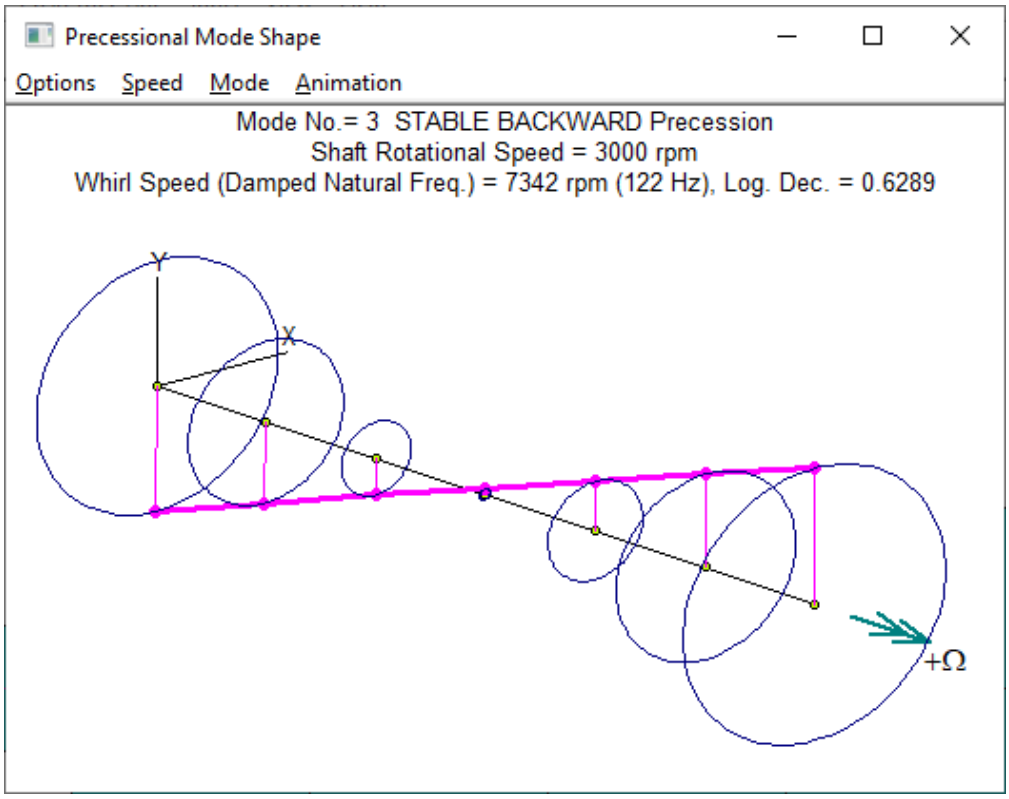
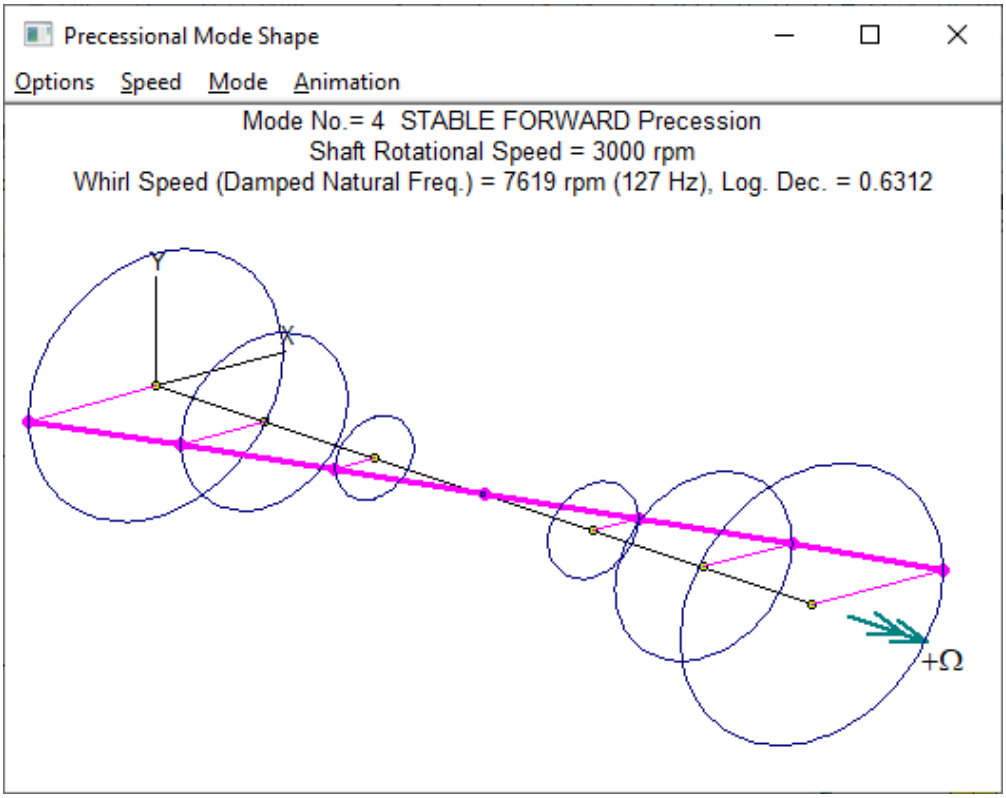


Fig. 22 – Example 3




Precessional Mode Shape
Options Speed Mode Animation
Mode No.= 2 STABLE FORWARD Precession
Shaft Rotational Speed = 3000 rpm
Whirl Speed (Damped Natural Freq.) = 4358 rpm (73 Hz), Log. Dec. = 0.4695
Y
X
+Ω


Precessional Mode Shape
Options Speed Mode Animation
Mode No.= 3 STABLE BACKWARD Precession
Shaft Rotational Speed = 3000 rpm
Whirl Speed (Damped Natural Freq.) = 7342 rpm (122 Hz), Log. Dec. = 0.6289
Y
X
+Ω



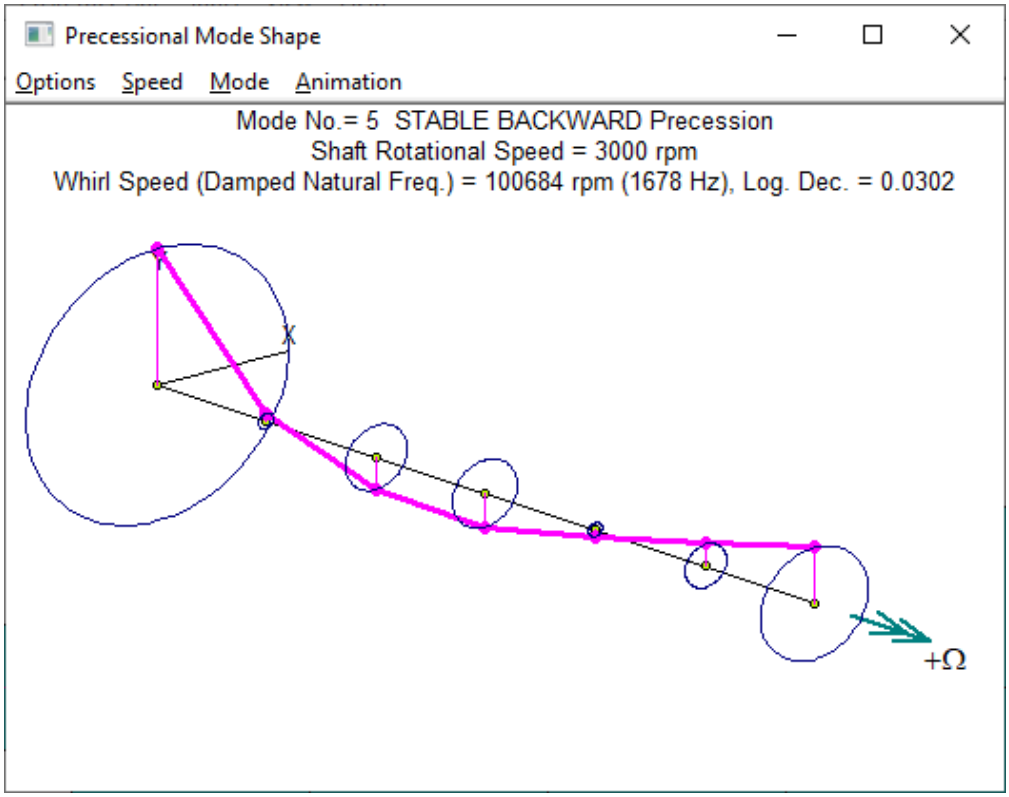


Fig. 23 – The first five modes at 3,000 rpm

Case 1: BaseMotion_3a.rot

A single base is assumed in this case. The base has a motion in Y direction and the excitation frequency is from 200 to 10,000 cpm with an increment of 50 cpm. This excitation frequency will excite the first two system natural frequencies and far below the third natural frequency. The base motion and the analysis input are shown below:

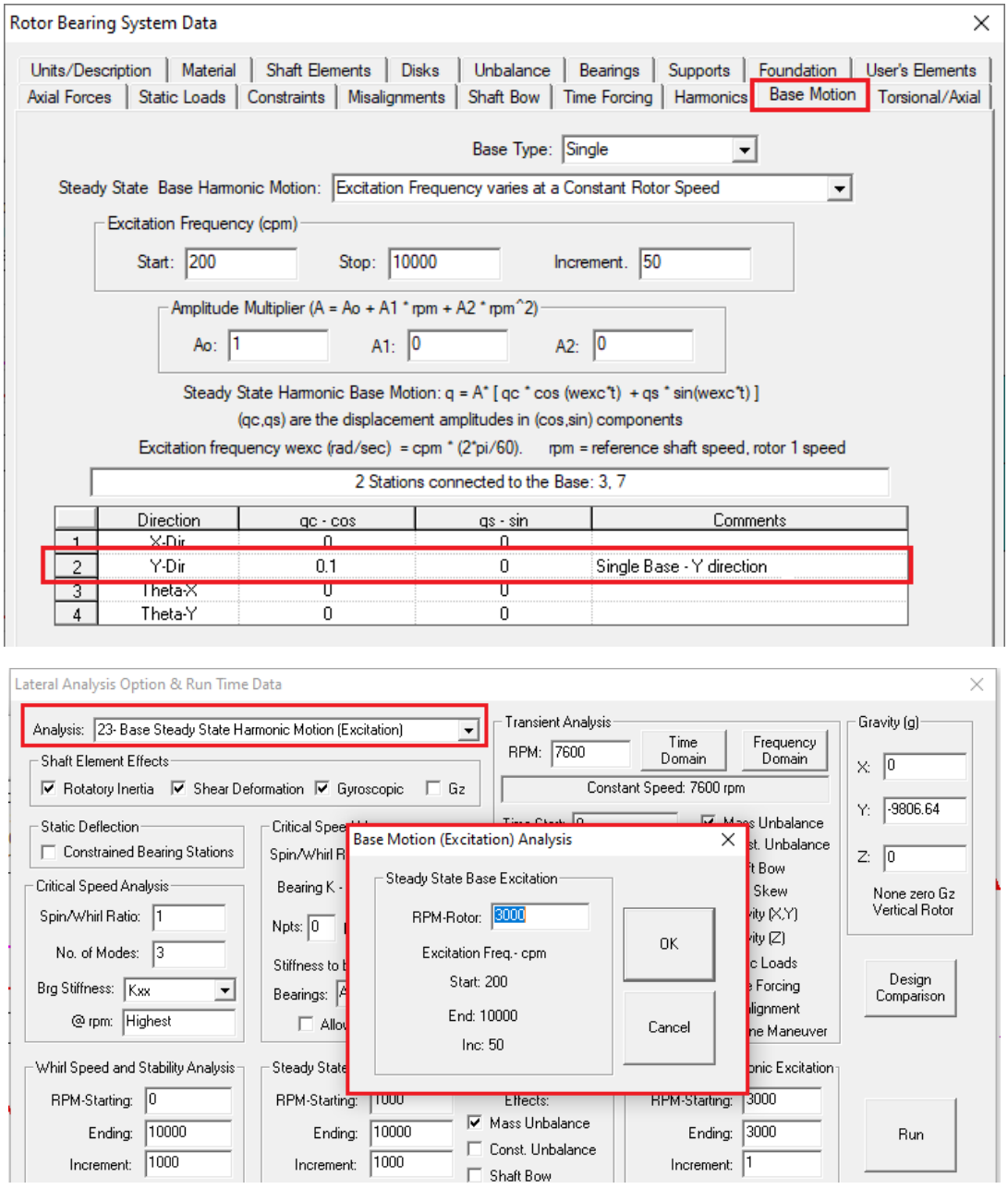
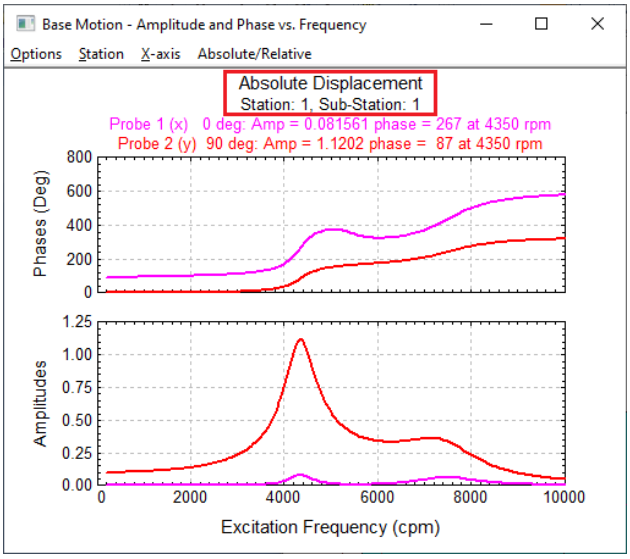
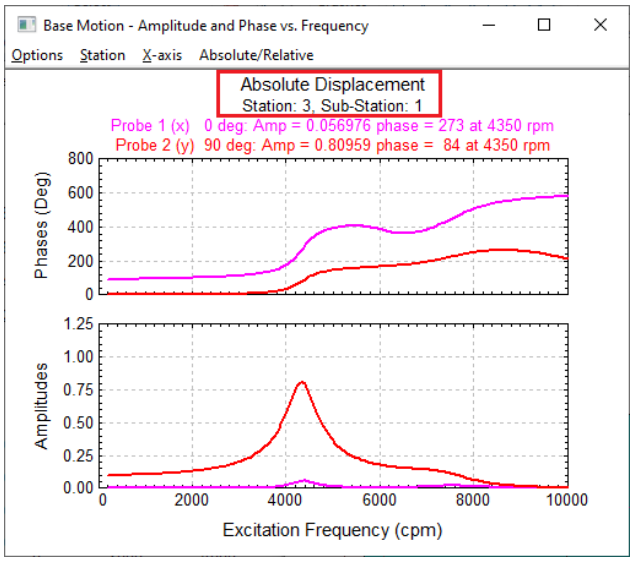
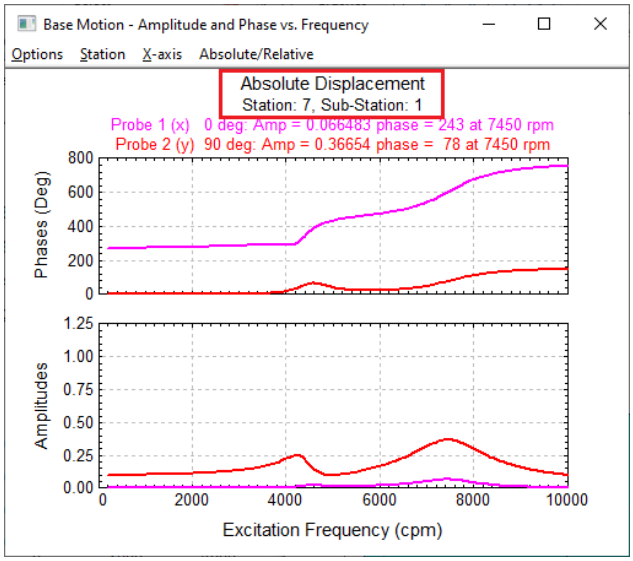


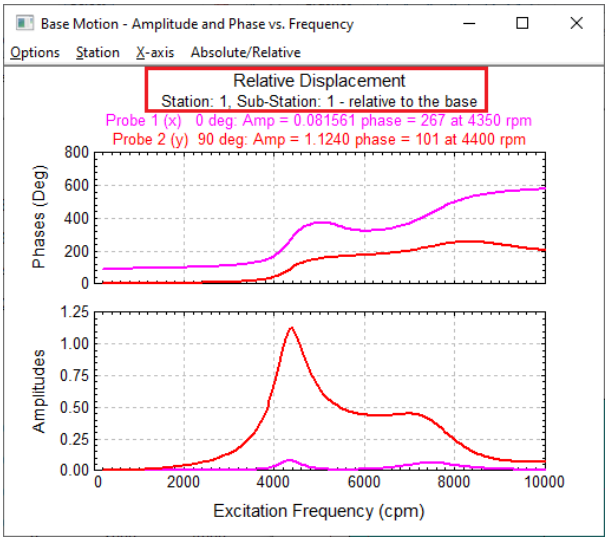
Fig. 24 – The Base Motion and Analysis Input

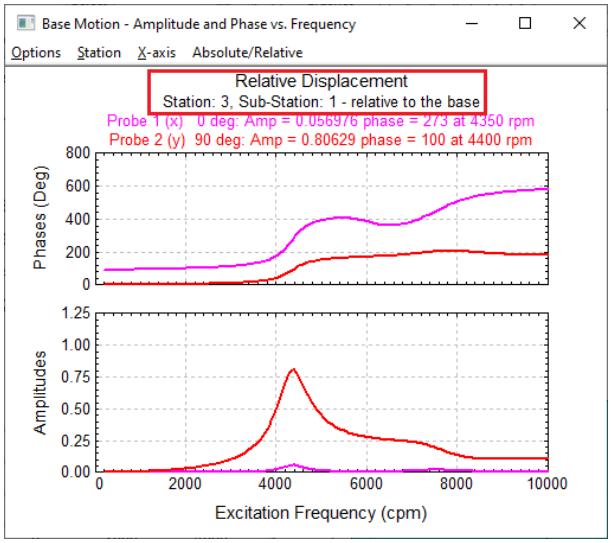
The displacements (absolute and relative) at the disk (station 1) and bearings (stations 3 and 7) are shown in Fig. 25 below:


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 1, Sub-Station: 1
Probe 1 (x) 0 deg: Amp = 0.081561 phase = 267 at 4350 rpm
Probe 2 (y) 90 deg: Amp = 1.1202 phase = 87 at 4350 rpm
Phases (Deg)
Amplitudes
Excitation Frequency (cpm)


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 3, Sub-Station: 1
Probe 1 (x) 0 deg: Amp = 0.056976 phase = 273 at 4350 rpm
Probe 2 (y) 90 deg: Amp = 0.80959 phase = 84 at 4350 rpm
Phases (Deg)
Amplitudes
Excitation Frequency (cpm)


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 7, Sub-Station: 1
Probe 1 (x) 0 deg: Amp = 0.066483 phase = 243 at 7450 rpm
Probe 2 (y) 90 deg: Amp = 0.36654 phase = 78 at 7450 rpm
Phases (Deg)
Amplitudes
Excitation Frequency (cpm)





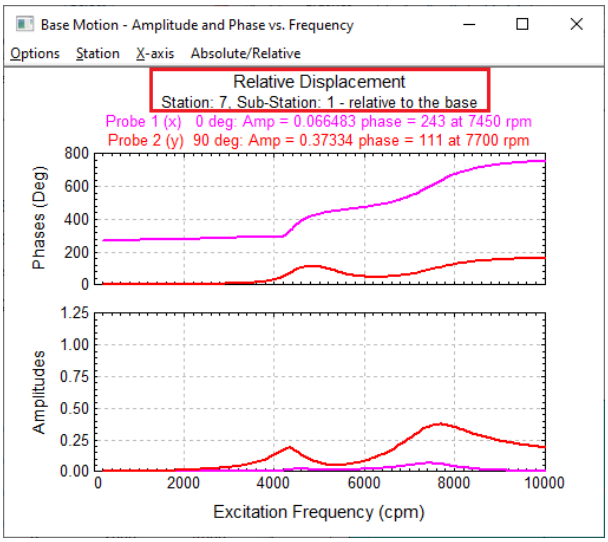


Fig. 25 – The Results for Base Motion

Forces transmitted through bearings to the base are:

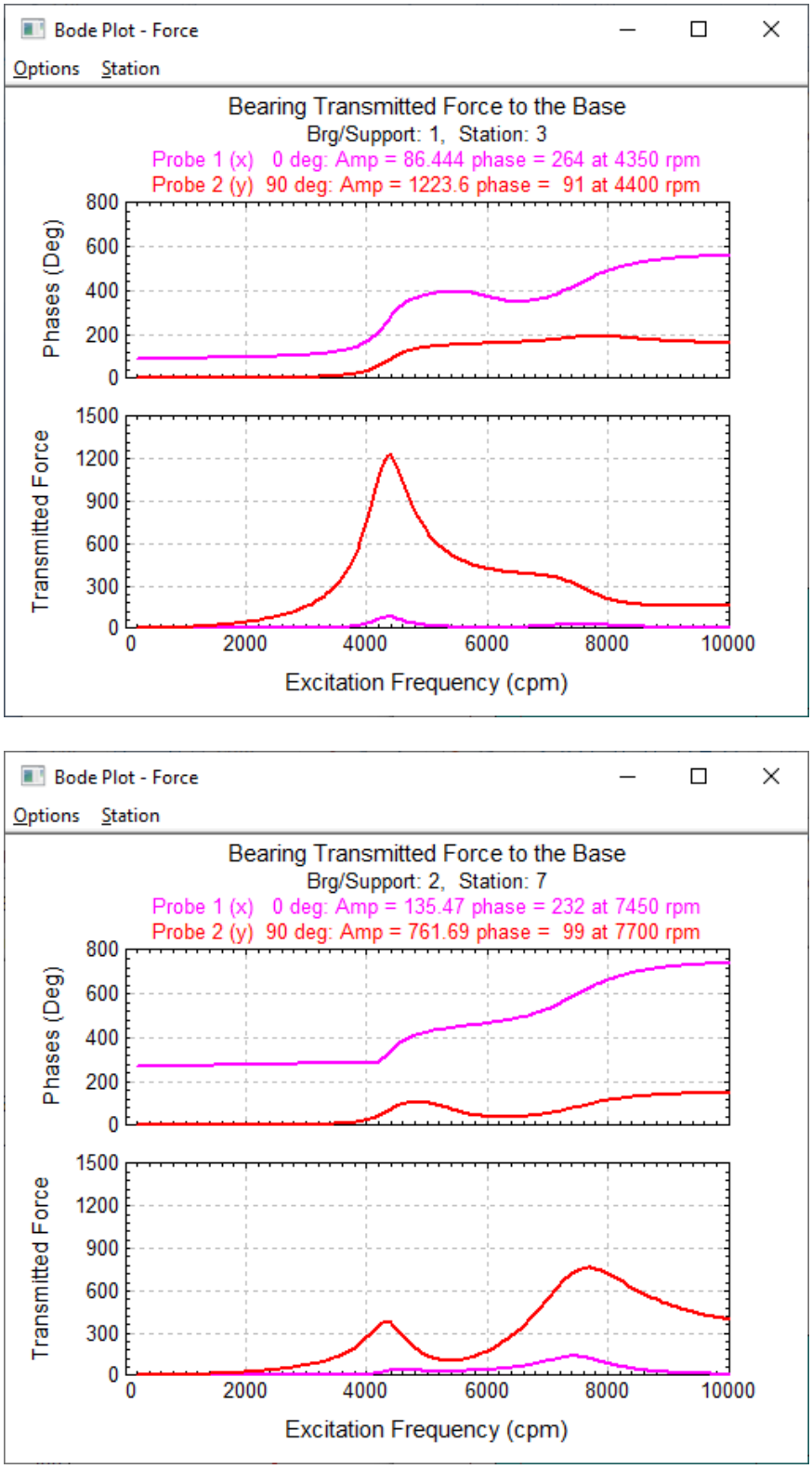
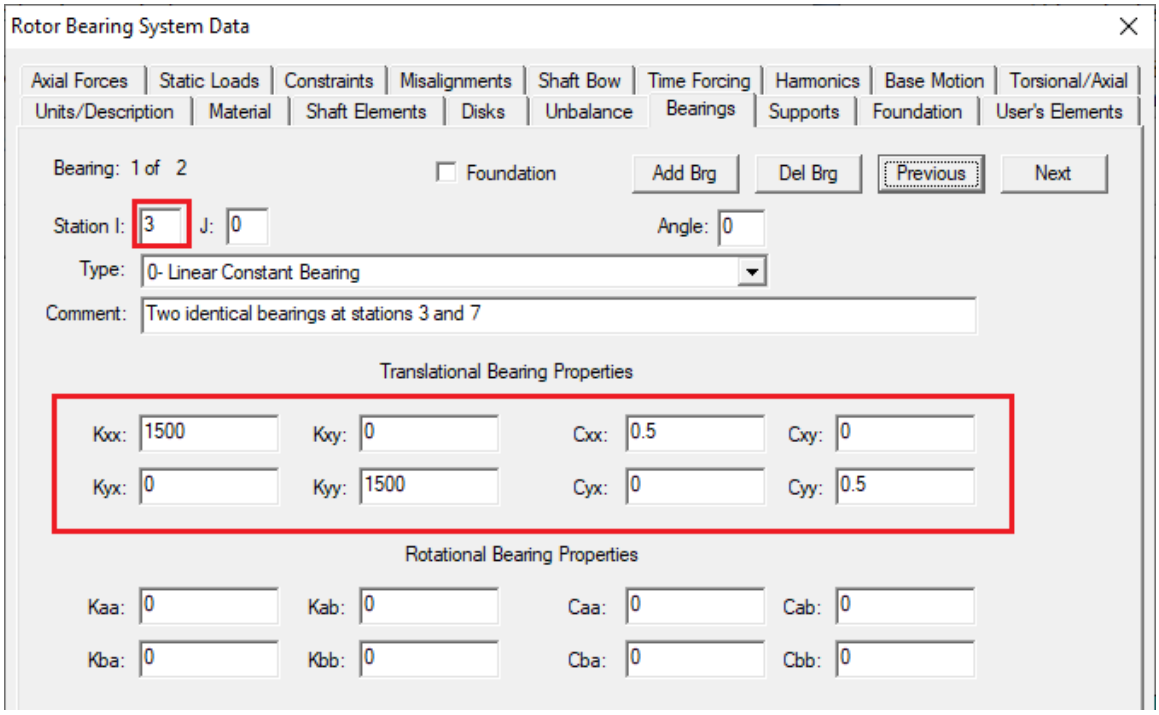
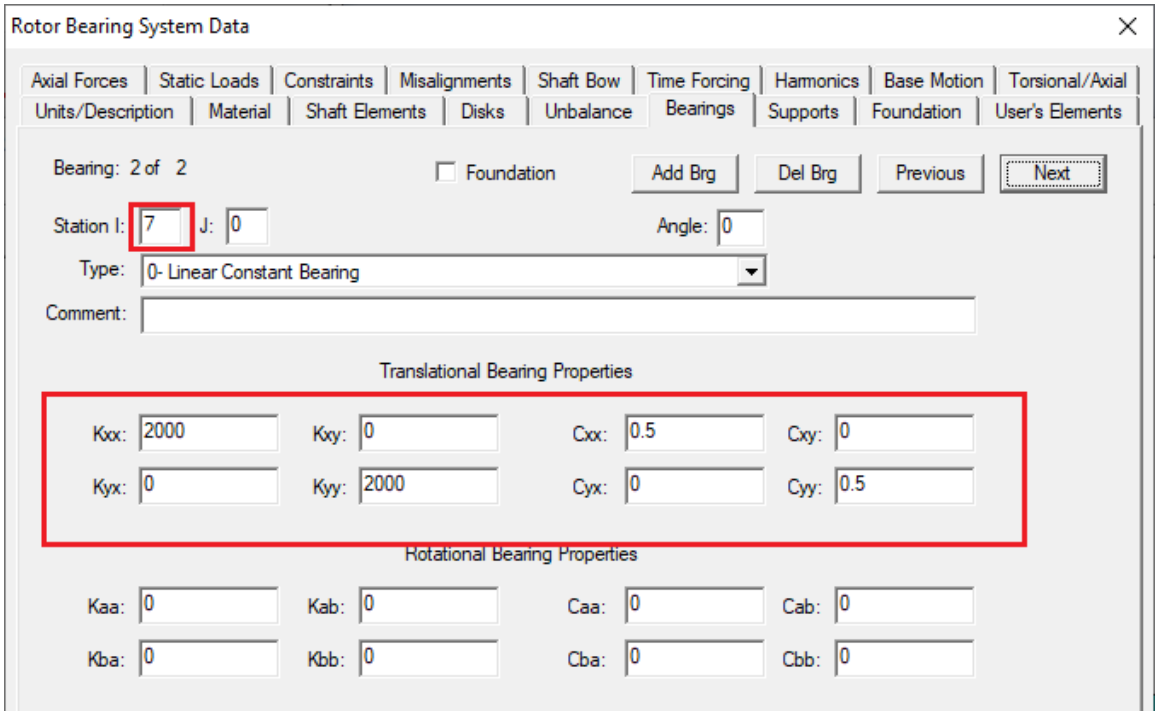


Fig. 26 – The Force Transmitted to the Base

Although the bearings are isotropic and uncoupled in the X and Y directions in this example, as shown below, the X and Y motions are coupled through the gyroscopic effect. So, even with the base motion in the Y direction only, the responses occur at both X and Y directions with dominant Y displacement.





Again, the absolute displacements can be verified using the Steady State Harmonic Excitation Analysis as described before. For comparison purposes, the excitation frequency at 4,350 cpm is selected. The excitation forces can then be calculated and entered in the harmonic excitation input, as shown below. The analysis is run at a constant rotor speed of 3,000 rpm. The results are also listed below.

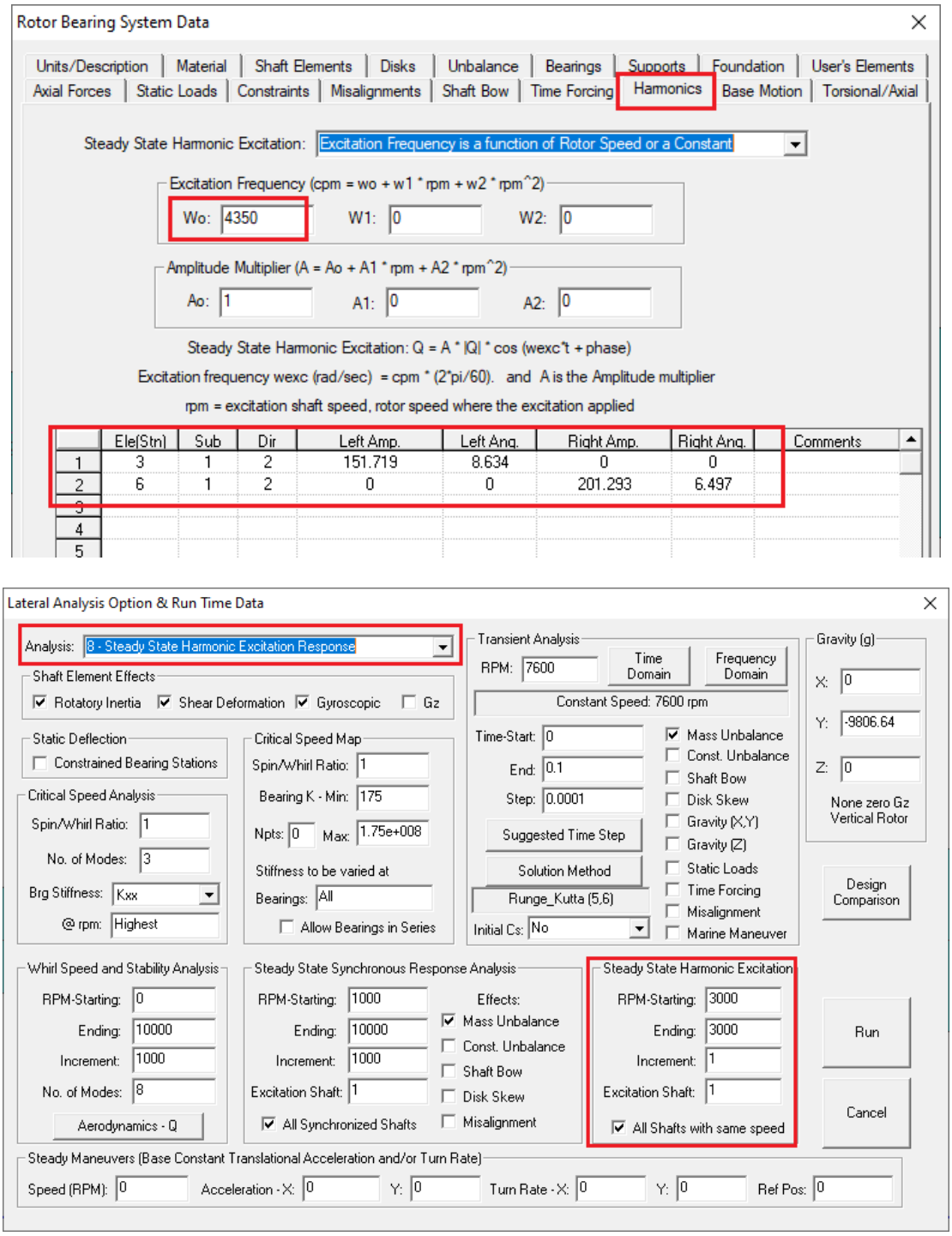


Fig. 27 – The Steady State Harmonic Excitation

```
************** Harmonic Response due to Shaft (1) Excitation ***************
               *** Excitation Frequency =  4350.0      cpm ***

   Shaft  1     Speed=    3000.00 rpm  =    314.16 R/S  =      50.00 Hz

 ********************** Shaft Element Displacements ************************
          =====  X  =====    =====  Y  =====        Elliptical Orbit Data
  stn sub Amplitude Phase    Amplitude Phase        A          B         G

   1   1  0.816E-01  267.5   0.112E+01   87.4    0.112E+01   0.130E-03    94.
   2   1  0.692E-01  269.7   0.964E+00   86.0    0.966E+00   0.448E-02    94.
   3   1  0.570E-01  272.9   0.810E+00   84.1    0.812E+00   0.870E-02    94.
   4   1  0.451E-01  277.8   0.657E+00   81.4    0.659E+00   0.127E-01    94.
   5   1  0.338E-01  286.0   0.507E+00   77.0    0.508E+00   0.164E-01    93.
   6   1  0.239E-01  301.7   0.363E+00   69.0    0.363E+00   0.190E-01    92.
   7   1  0.177E-01  332.7   0.233E+00   51.5    0.233E+00   0.173E-01    89.

 ******************************************************************************

  Element Internal Shear Forces and Moments (Semi-Major Axis)
       ------------ Left -------------  ------------ Right -------------
  Ele       Shear          Moment            Shear          Moment

   1       81.574          322.01           187.76          9119.1
   2       187.76          9119.1           453.22          30367.
   3       773.18          30367.           553.88          13049.
   4       553.88          13049.           70.136          31195.
   5       70.136          31195.           291.51          22154.
   6       291.51          22154.           378.61        0.13147E-08

 ******************************************************************************

 ***************** Linear Bearing/Support Force and Moment ******************
   stn     =====  X  =====    =====  Y  =====        Elliptical Orbit Data
   I  J    Amplitude Phase    Amplitude Phase        A          B         G

   3  0   0.864E+02  264.3   0.123E+04   75.5    0.123E+04   0.132E+02    94.
 Moment    0.000E+00    0.0   0.000E+00    0.0    0.000E+00   0.000E+00     0.
   7  0   0.356E+02  326.2   0.470E+03   45.0    0.470E+03   0.349E+02    89.
 Moment    0.000E+00    0.0   0.000E+00    0.0    0.000E+00   0.000E+00     0.

 ******************************************************************************
```

The absolute displacements are identical to the results from the base motion analysis. However, the forces transmitted through bearings are not the same since the steady state harmonic excitation analysis did not include the base motion.

Case 2: BaseMotion_3b.rot

In this case, the Single Base is replaced by Multiple Base Option with the identical base motion, as shown in Fig. 28.

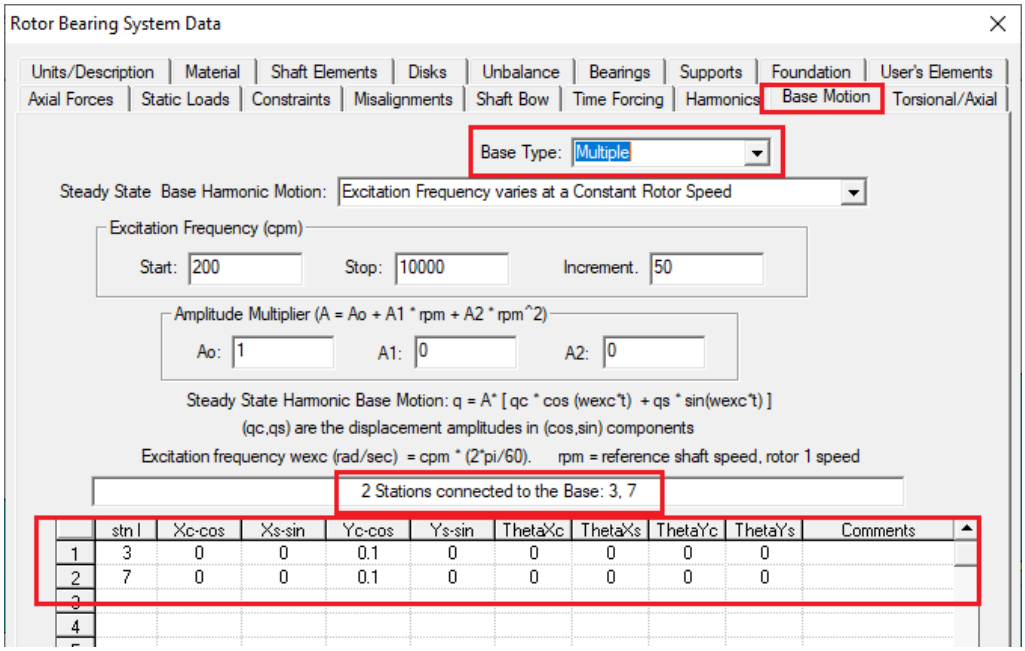


Fig. 28 – Multiple Base

The results are identical to the Case 1 and not presented here. However, when plotting the relative displacements, one must select which base will be used for the relative displacement as shown in Fig. 29.

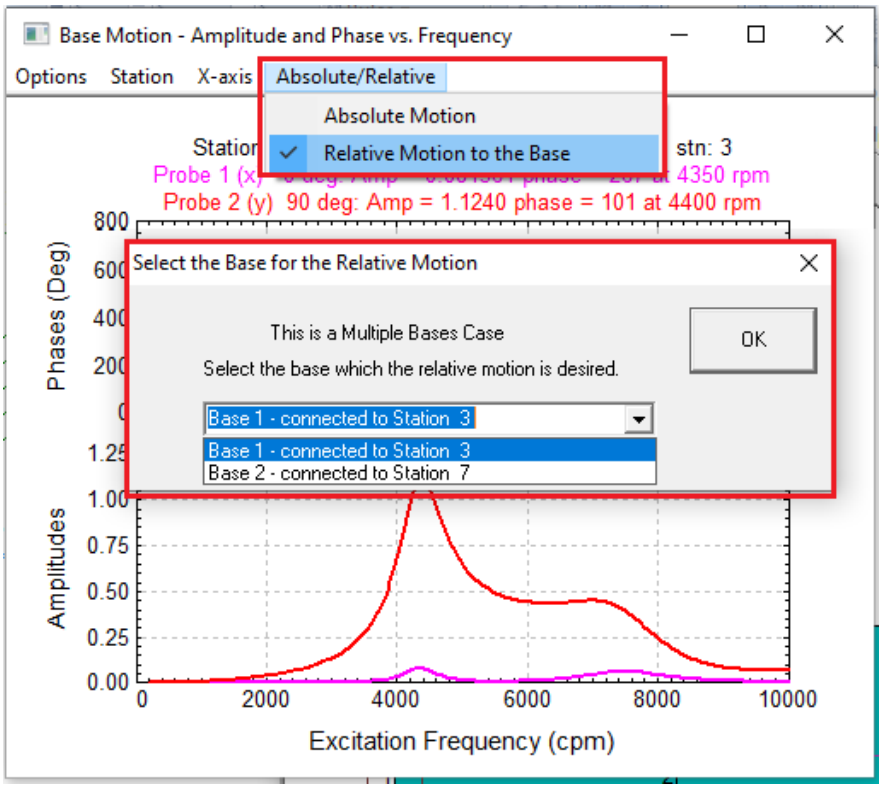


Fig. 29 – The Relative Displacement

Case 3: BaseMotion_3c.rot

In this case, both bearings are subject to different base motion, as shown in Fig. 30. Note that the base motion can have different amplitude and phase, but the same frequency. The first bearing (station 3) has a base motion in the Y direction only. The second bearing (station 7) has the base motion in both X and Y directions.

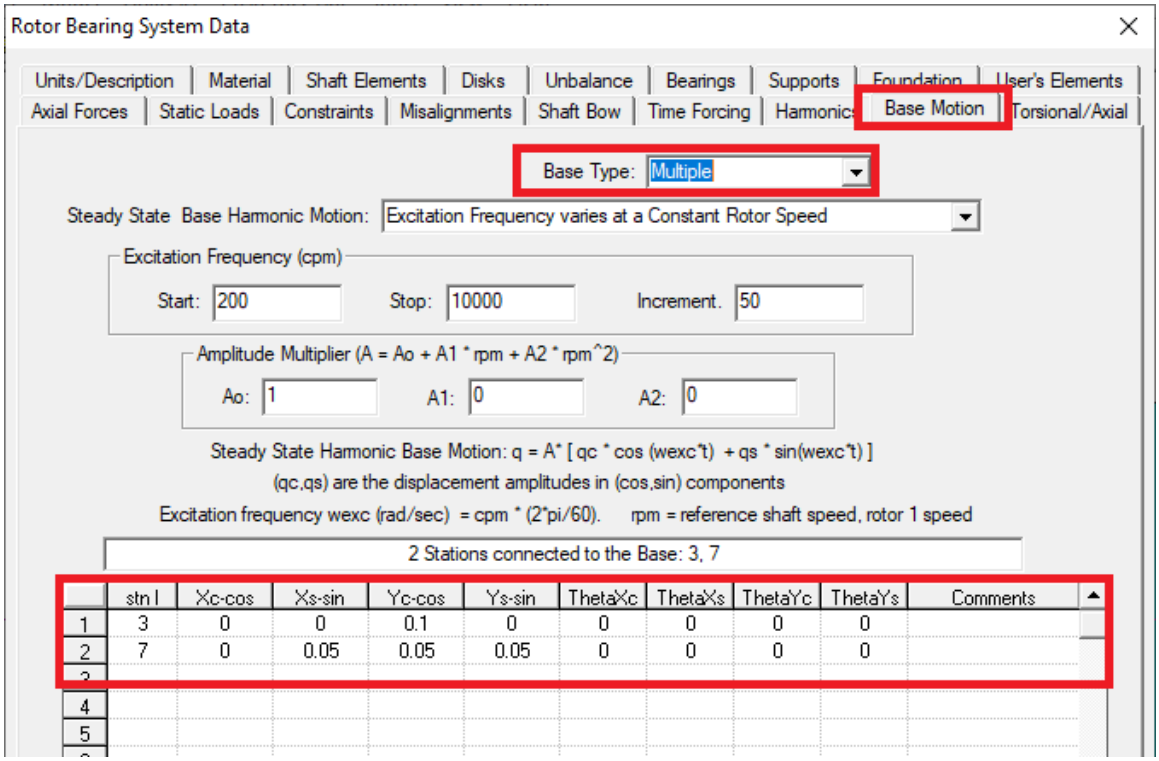
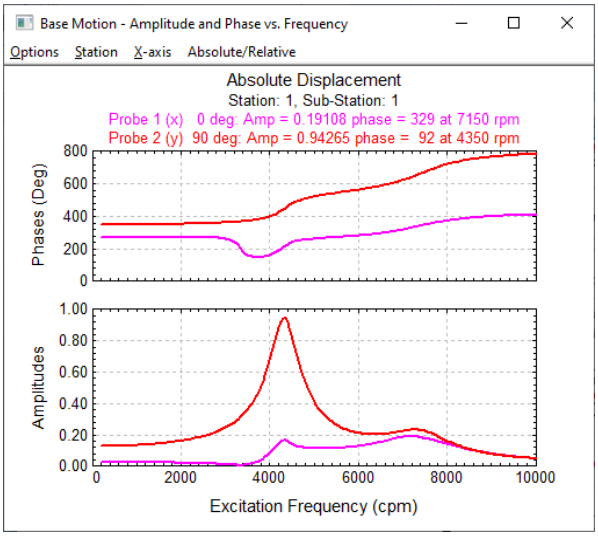
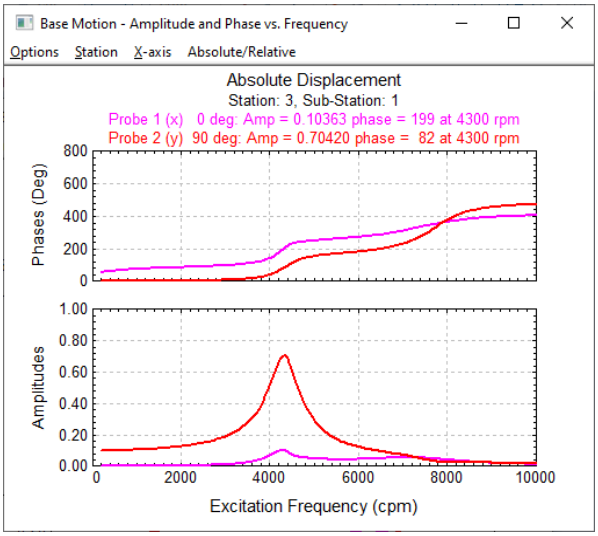


Fig. 30 – The Base Motion

The absolute displacements for the disk and both bearings are shown in Fig. 31. The relative motions for both bearings are shown in Fig. 32. The forces transmitted through bearings are shown in Fig. 33.





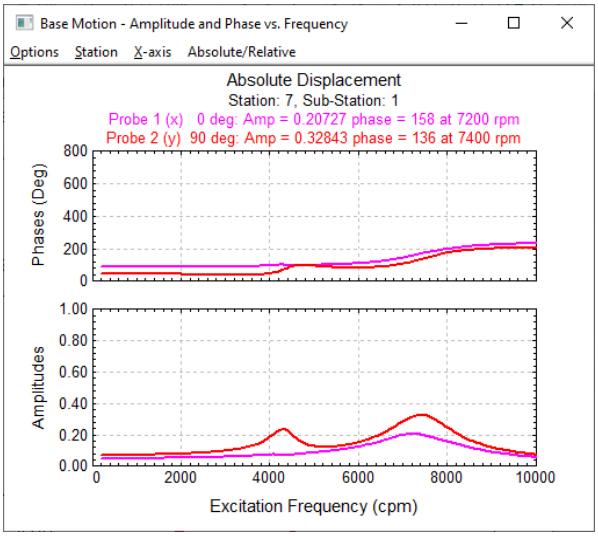


Fig. 31 – The Absolute Displacements

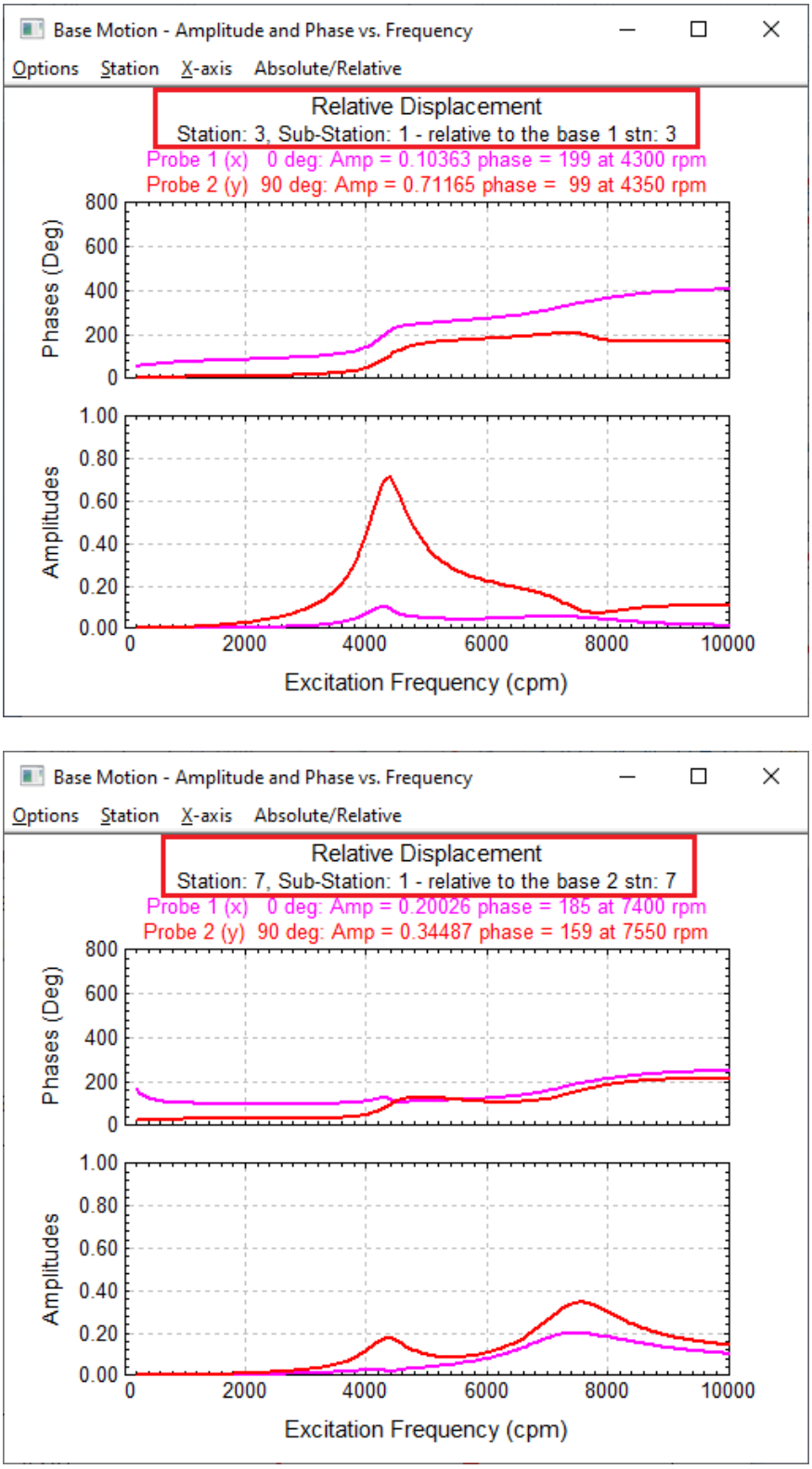


Fig. 32 – The Relative Displacements

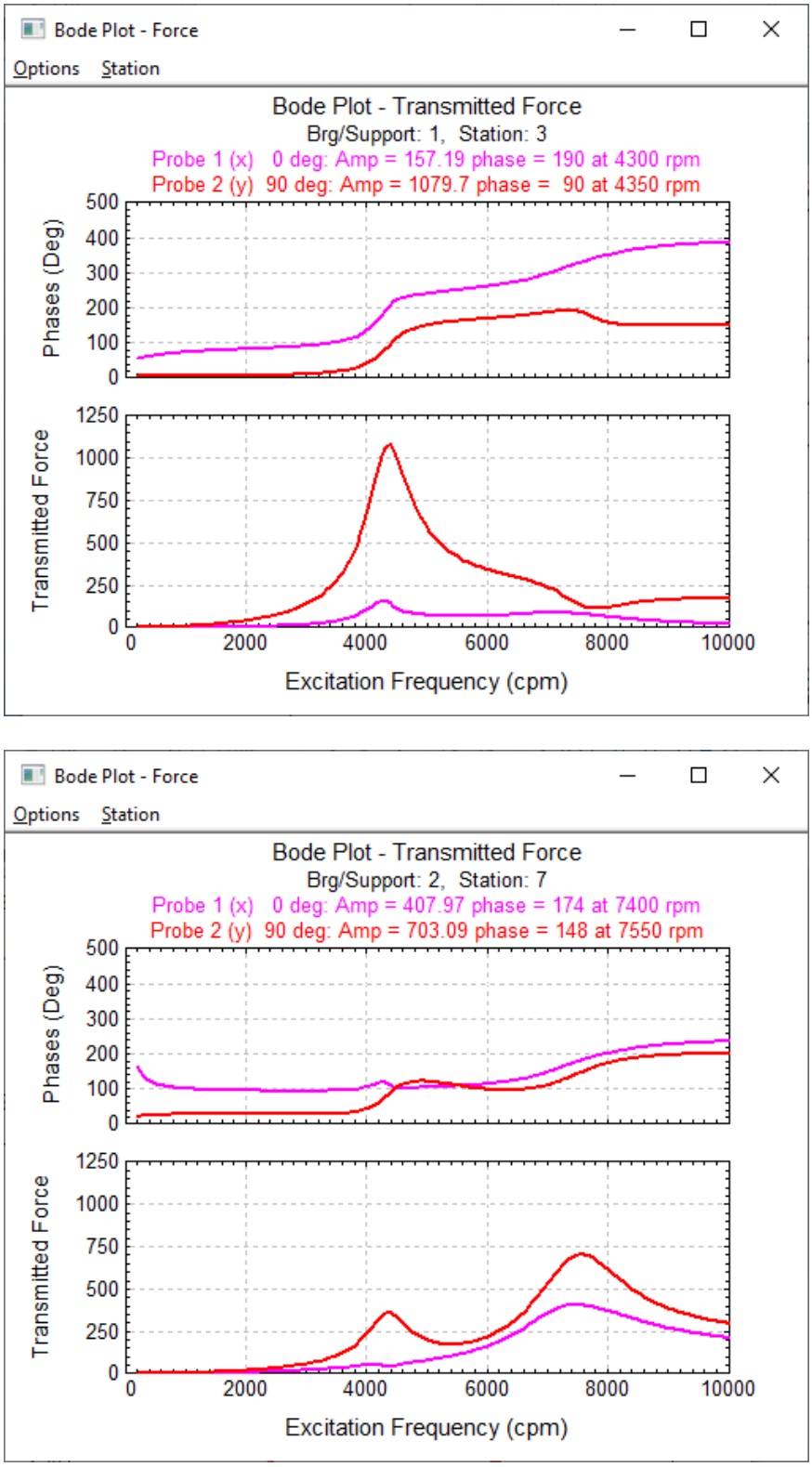


Fig. 33 – The Bearing Transmitted Forces

To verify the absolute displacements caused by the base motion, the steady state harmonic excitation analysis is again used. For comparison purposes, the frequency at 4,350 cpm is selected. The amplitudes and phases are entered accordingly as shown in Fig. 34.

The right hand side of the equation for the Steady State Harmonic Excitation:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{Bmatrix} z_x \\ z_y \end{Bmatrix} + \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{Bmatrix} \dot{z}_x \\ \dot{z}_y \end{Bmatrix} \tag{13}$$

where

$$\begin{Bmatrix} z_x \\ z_y \end{Bmatrix} = \begin{Bmatrix} x_c \\ y_c \end{Bmatrix} \cos(\omega_{exc} t) + \begin{Bmatrix} x_s \\ y_s \end{Bmatrix} \sin(\omega_{exc} t)$$

$$\begin{Bmatrix} \dot{z}_x \\ \dot{z}_y \end{Bmatrix} = \omega_{exc} \left(\begin{Bmatrix} x_s \\ y_s \end{Bmatrix} \cos(\omega_{exc} t) - \begin{Bmatrix} x_c \\ y_c \end{Bmatrix} \sin(\omega_{exc} t) \right)$$

and

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} F_{xc} \\ F_{xc} \end{Bmatrix} \cos(\omega_{exc} t) + \begin{Bmatrix} F_{xs} \\ F_{ys} \end{Bmatrix} \sin(\omega_{exc} t)$$

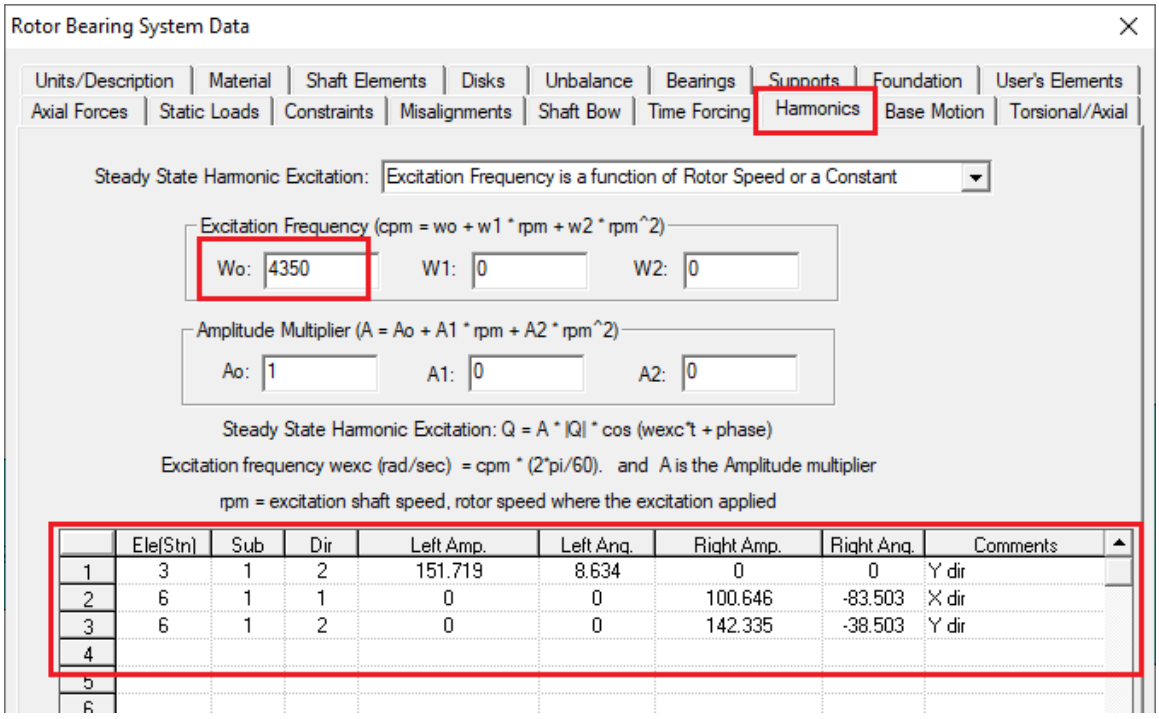


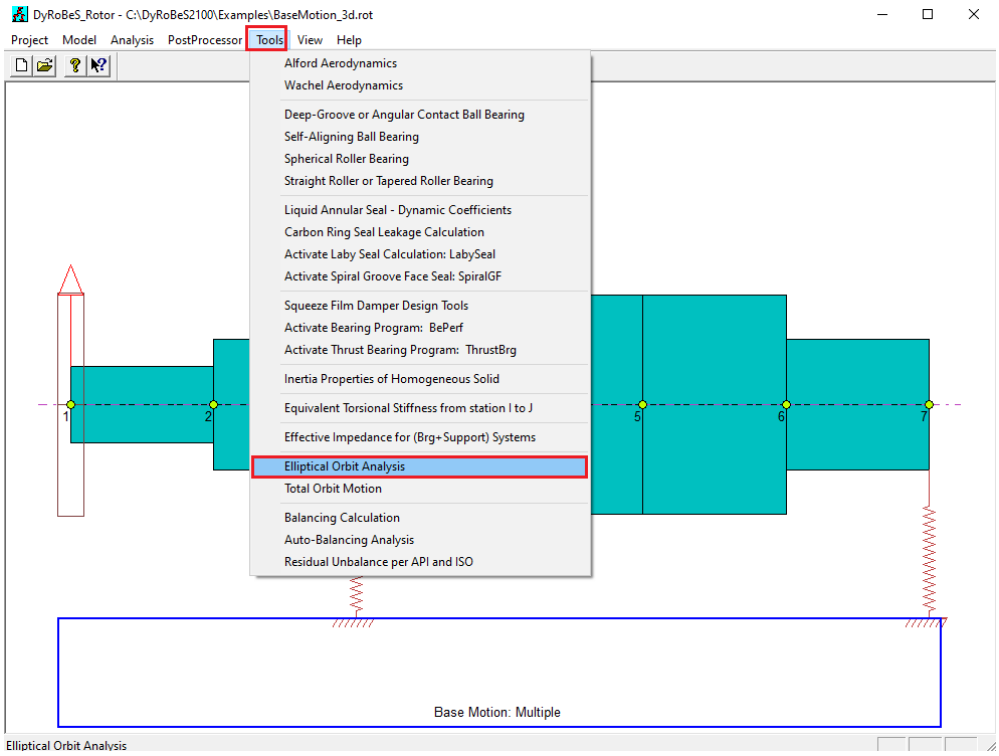
Fig. 34 – The Steady State Harmonic Excitation

Since the comparison is selected only in one frequency, the results are compared as below:

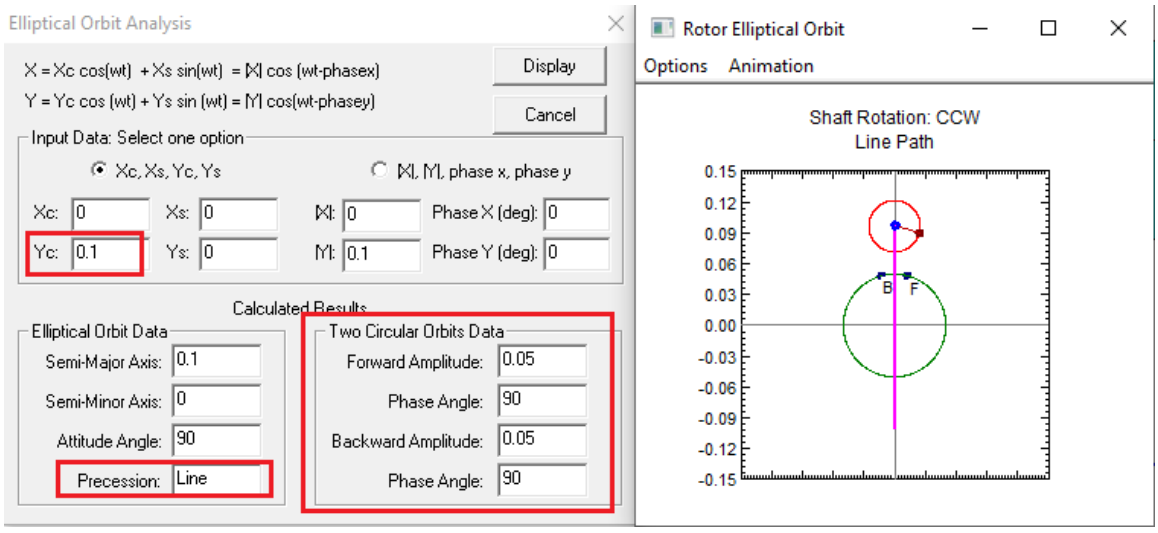
```
************** Harmonic Response due to Shaft (1) Excitation ***************
              *** Excitation Frequency =  4350.0      cpm ***

  Shaft  1     Speed=    3000.00 rpm  =    314.16 R/S  =      50.00 Hz

*********************** Shaft Element Displacements ************************
         =====  X  =====    =====  Y  =====        Elliptical Orbit Data
 stn sub Amplitude Phase    Amplitude Phase        A          B         G

  1   1  0.165E+00  220.9   0.943E+00   92.1    0.948E+00  -0.128E+00    96.
  2   1  0.132E+00  216.6   0.823E+00   91.5    0.826E+00  -0.107E+00    95.
  3   1  0.100E+00  209.5   0.704E+00   90.6    0.705E+00  -0.873E-01    94.
  4   1  0.711E-01  196.2   0.586E+00   89.3    0.586E+00  -0.680E-01    92.
  5   1  0.504E-01  169.1   0.468E+00   87.5    0.468E+00  -0.498E-01    89.
  6   1  0.495E-01  128.3   0.351E+00   84.4    0.352E+00  -0.341E-01    84.
  7   1  0.692E-01   99.8   0.235E+00   78.2    0.244E+00  -0.246E-01    75.

*******************************************************************************


************ Harmonic Response due to Base Motion (Excitation) *************
              *** Excitation Frequency =  4350.0      cpm ***

  Shaft  1     Speed=    3000.00 rpm  =    314.16 R/S  =      50.00 Hz

*********************** Shaft Element Displacements ************************
         =====  X  =====    =====  Y  =====        Elliptical Orbit Data
 stn sub Amplitude Phase    Amplitude Phase        A          B         G

  1   1  0.165E+00  220.9   0.943E+00   92.1    0.948E+00  -0.128E+00    96.
  2   1  0.132E+00  216.6   0.823E+00   91.5    0.826E+00  -0.107E+00    95.
  3   1  0.100E+00  209.5   0.704E+00   90.6    0.705E+00  -0.873E-01    94.
  4   1  0.711E-01  196.2   0.586E+00   89.3    0.586E+00  -0.680E-01    92.
  5   1  0.504E-01  169.1   0.468E+00   87.5    0.468E+00  -0.498E-01    89.
  6   1  0.495E-01  128.3   0.351E+00   84.4    0.352E+00  -0.341E-01    84.
  7   1  0.692E-01   99.8   0.235E+00   78.2    0.244E+00  -0.246E-01    75.

*******************************************************************************
```

Whirling Direction

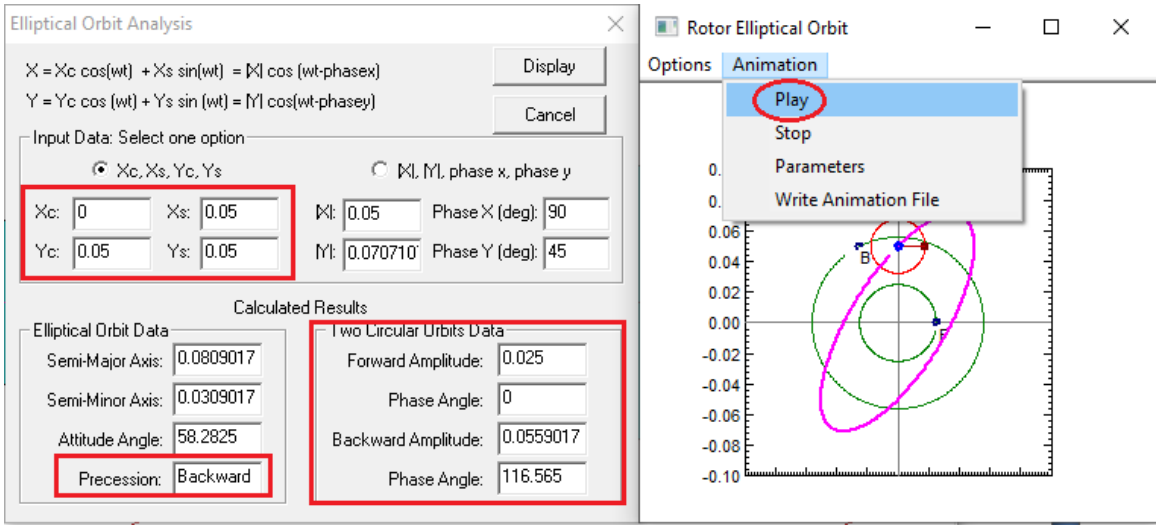
To fully understand the effect of the base motion, we need to also examine the precessions of the base motion and the rotor motion. In this Case 3, the base motion at station 3 is a straight line motion which can excite both forward and backward whirling modes. The base motion at station 7 is a backward precessional elliptical motion which tends to excite the backward precessional modes more. To view the base motion, there is a Tool in DyRoBeS which can help you visualize the base motion, as shown below:



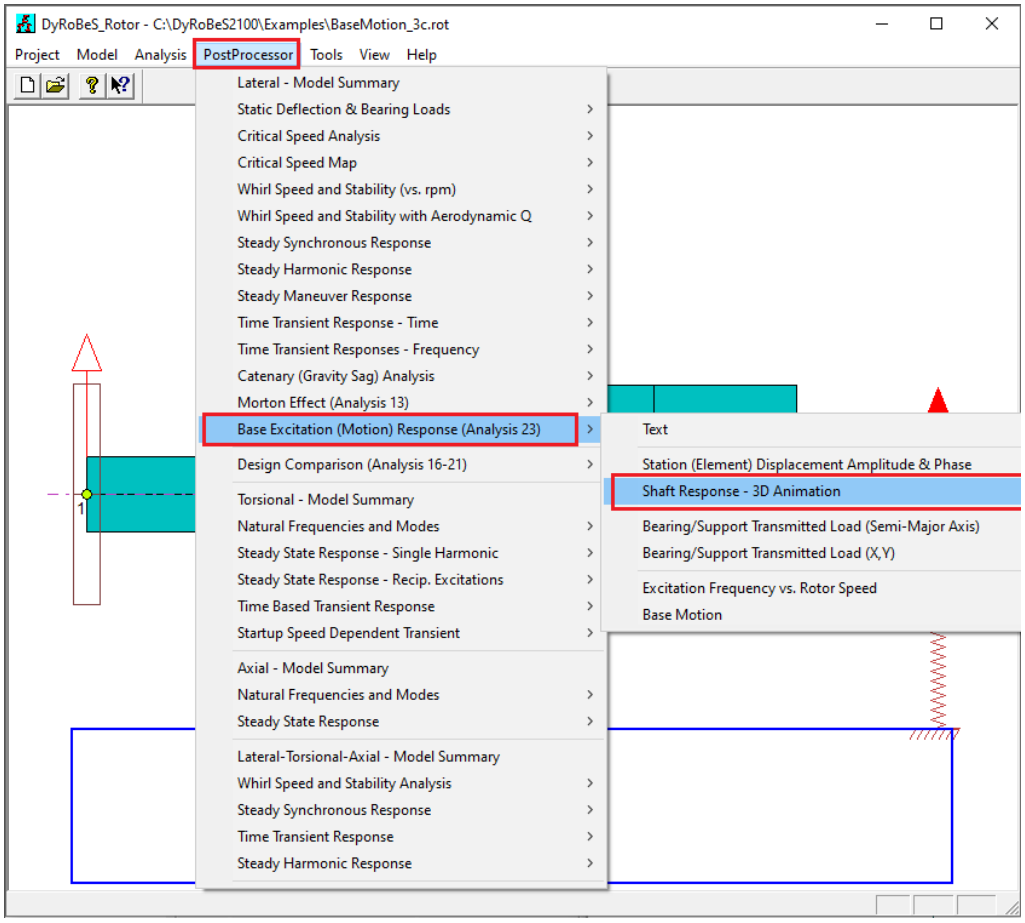
Base Motion at Station 3:

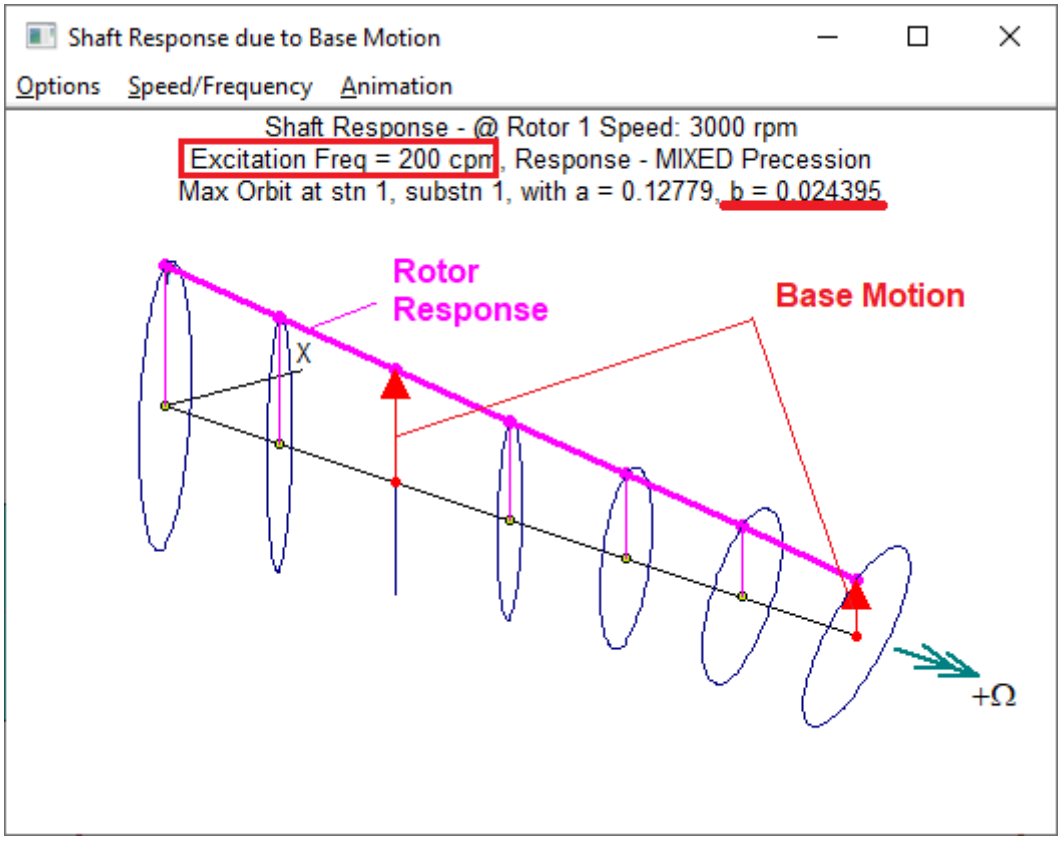


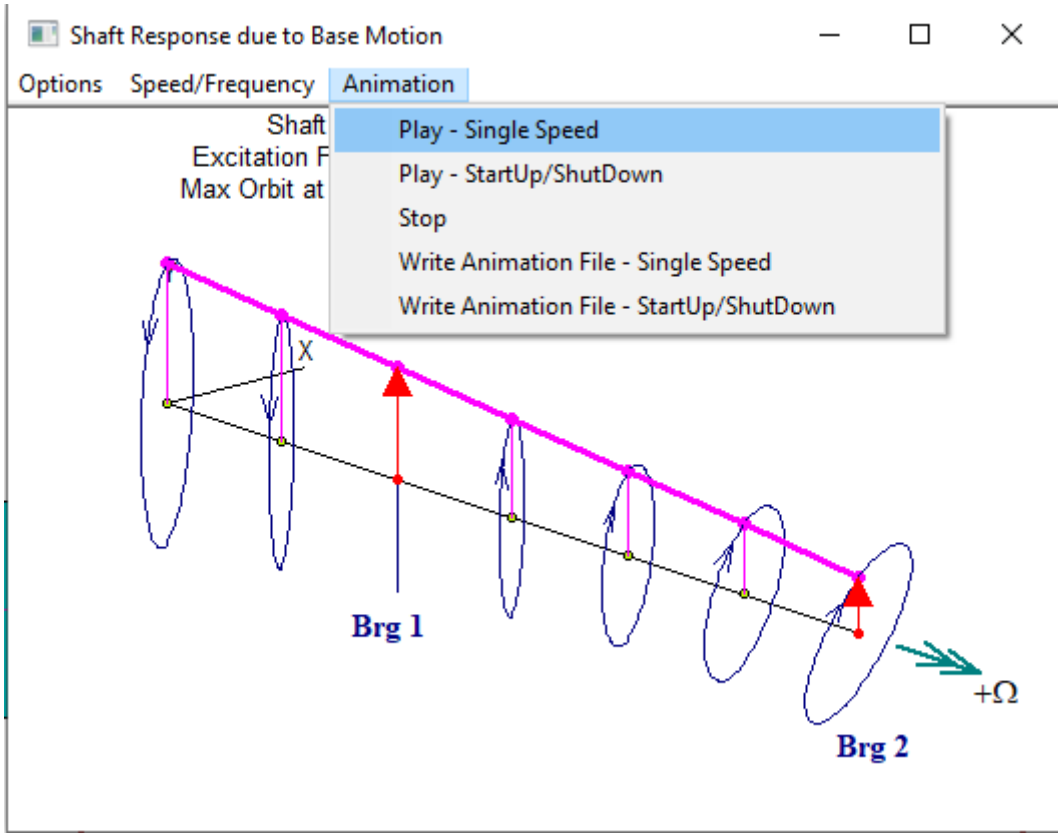
Base Motion at Station 7:



Use the Animation Play option, you can visualize the motion better. Now, let us go back to examine the entire rotor response (absolute displacements) from the Base Motion Analysis at the excitation frequency of 200, 4312, 4358, 7342, and 7619 cpm.

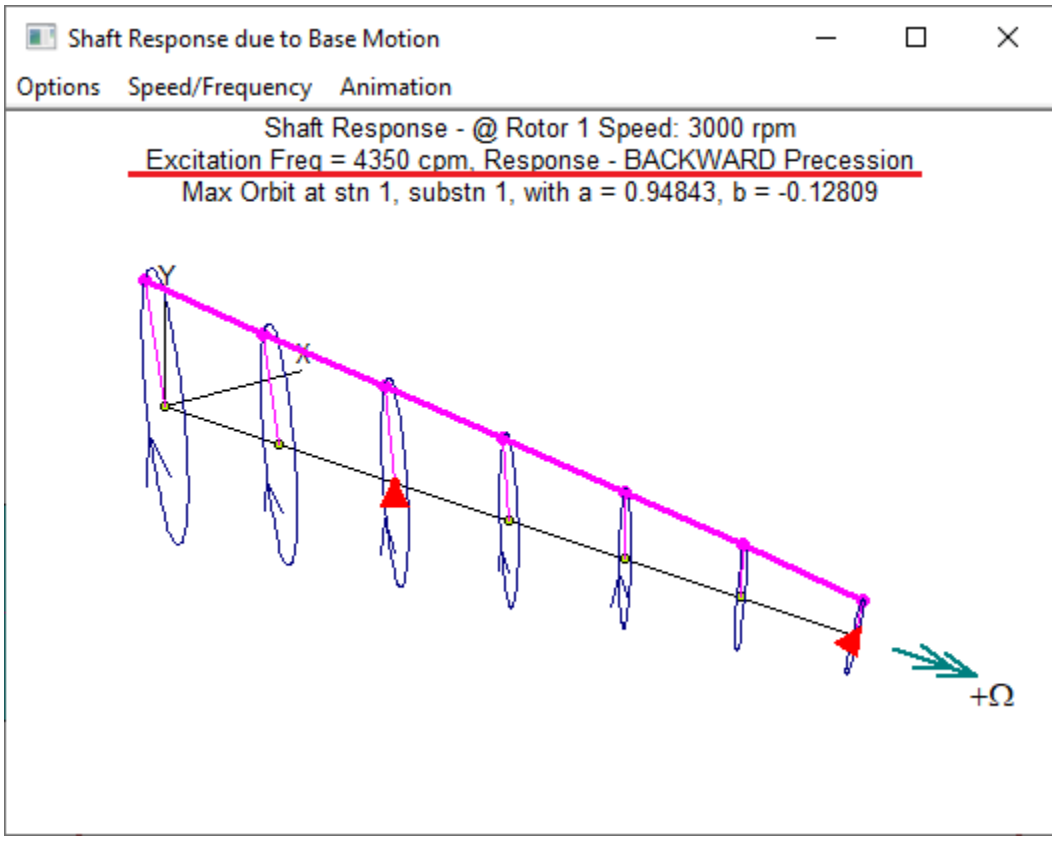



Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 200 cpm, Response - MIXED Precession
Max Orbit at stn 1, substn 1, with a = 0.12779, b = 0.024395
Rotor Response
Base Motion
X
+Ω

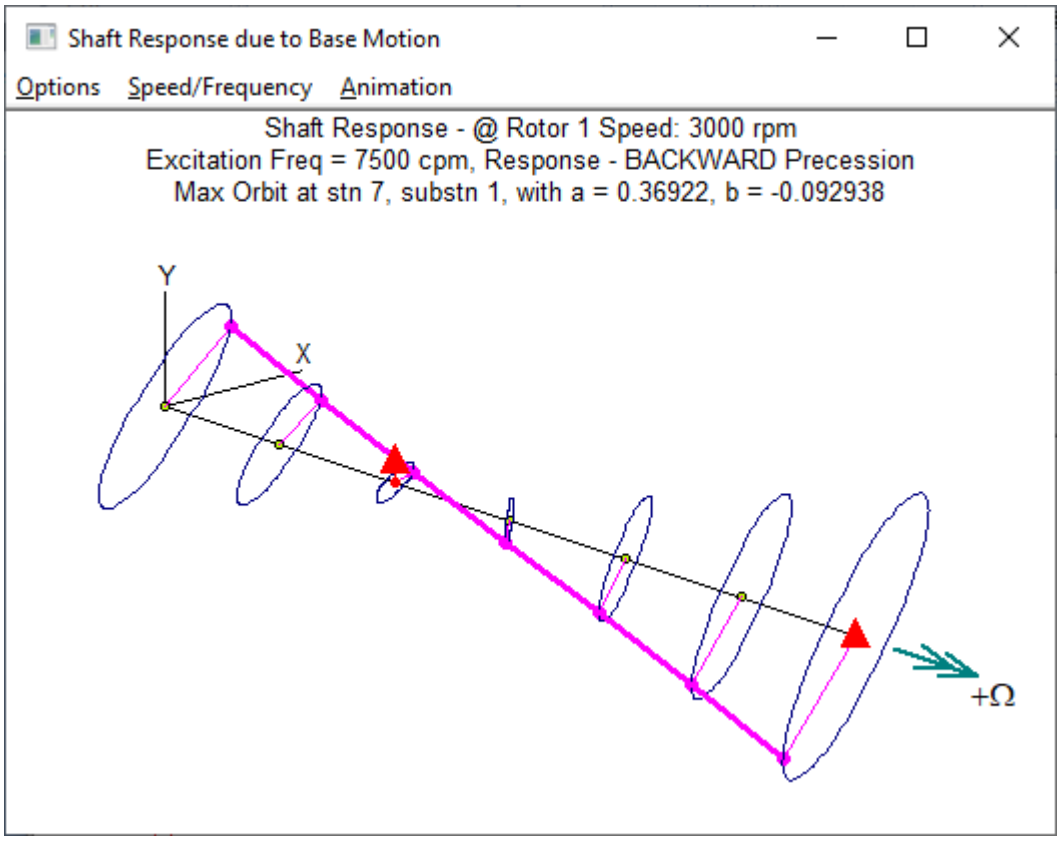

Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Play - Single Speed
Play - StartUp/ShutDown
Stop
Write Animation File - Single Speed
Write Animation File - StartUp/ShutDown
X
Brg 1
Brg 2
+Ω

In the graph, the properties of the max orbit are printed. The value a is the semi-major axis, and b is the semi-minor axis. The positive sign of the semi-minor axis indicates the orbit is a forward precession and negative sign indicates the backward precession. The **red arrow** represents the base motion. At very low excitation frequency (I.e., low frequency ratio), the rotor moves with the base and there is no relative movement between rotor and base. The rotor whirls in the forward precession for the stations 1 (max orbit) and 2. The station 3 is a straight line motion which is the same as the base motion. After the straight line motion, the rest of the rotor whirls in the backward precession, same as the station 7 base motion. So, the rotor is whirling in a so-called mixed precession.

At the excitation frequency of 4350 cpm, the entire rotor whirls in the backward precession at resonance. The rotor deflection shape is similar to the first mode. The rotor response is far larger than the base motion at or near resonance.

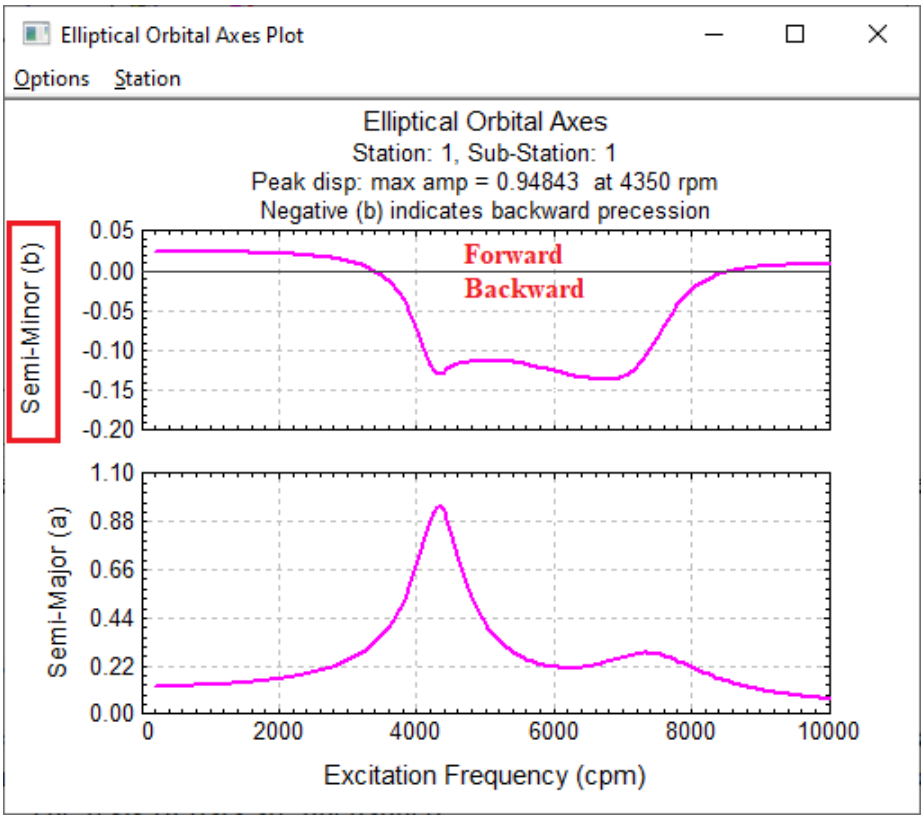


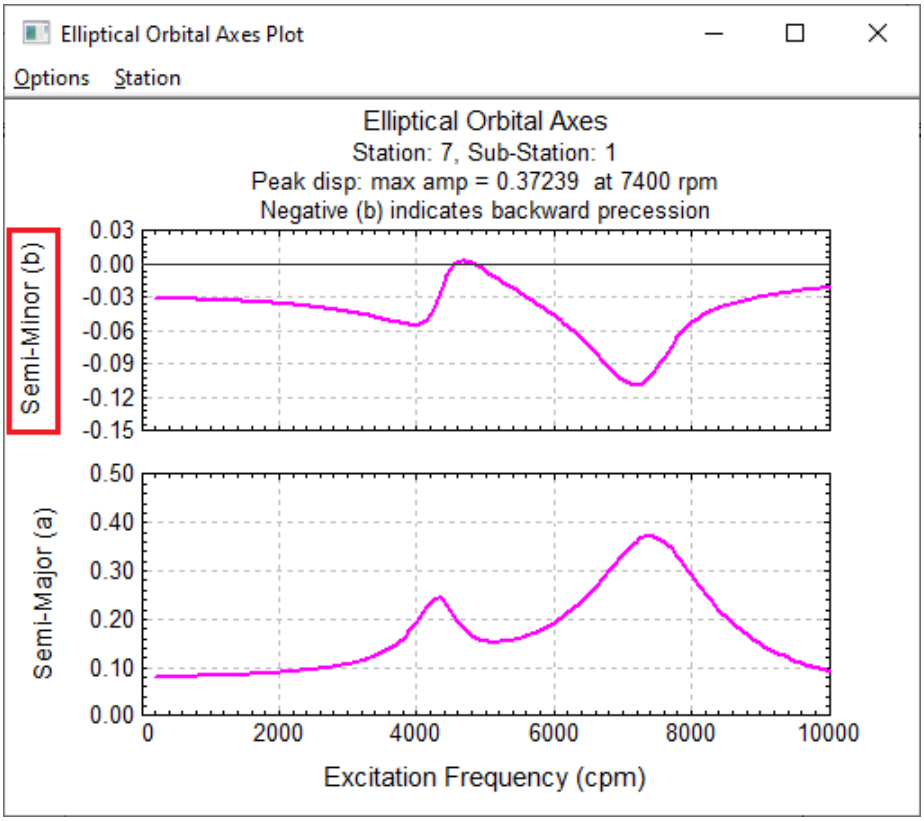
At the excitation frequency of 7500 cpm, the entire rotor whirls also in the backward precession. The rotor deflection shape is similar to the third mode.



To check the whirling direction for all the excitation frequency range, the easiest way is to plot the elliptical orbital axes (semi-major and semi-minor axes). The negative semi-minor axis indicates a backward whirl.

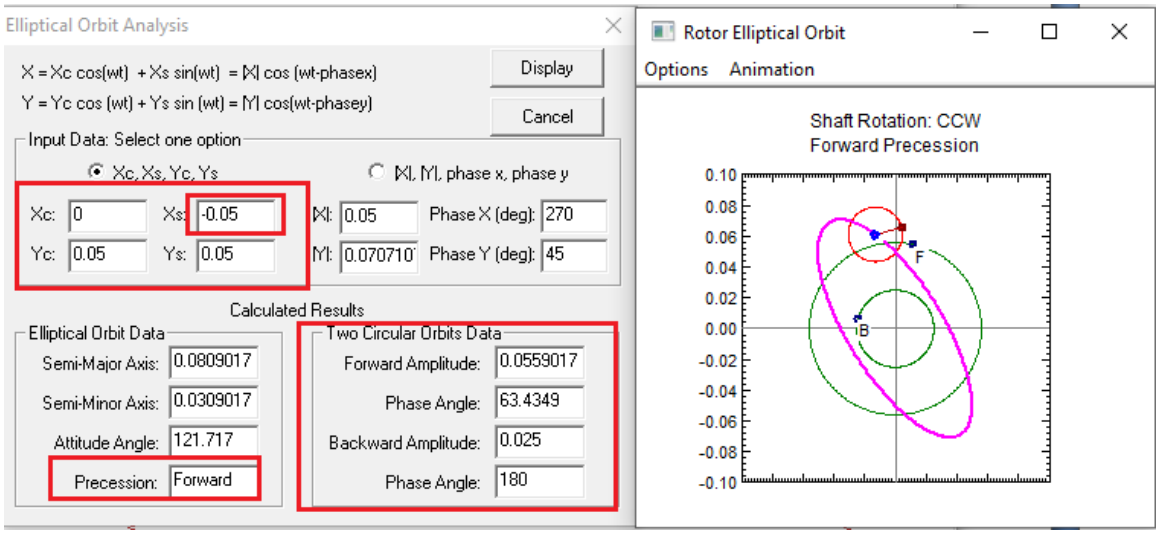
The orbital axes for the stations 1 and 7 are shown below. It shows that the station 1 starts with forward precessions at low excitation frequency and becomes backward precessions around 3400 cpm, then turns to forward precessions again after 8550 cpm. However, for station 7, it starts with backward precessions, and becomes forward precessions only between 4600 cpm and 4800 cpm.

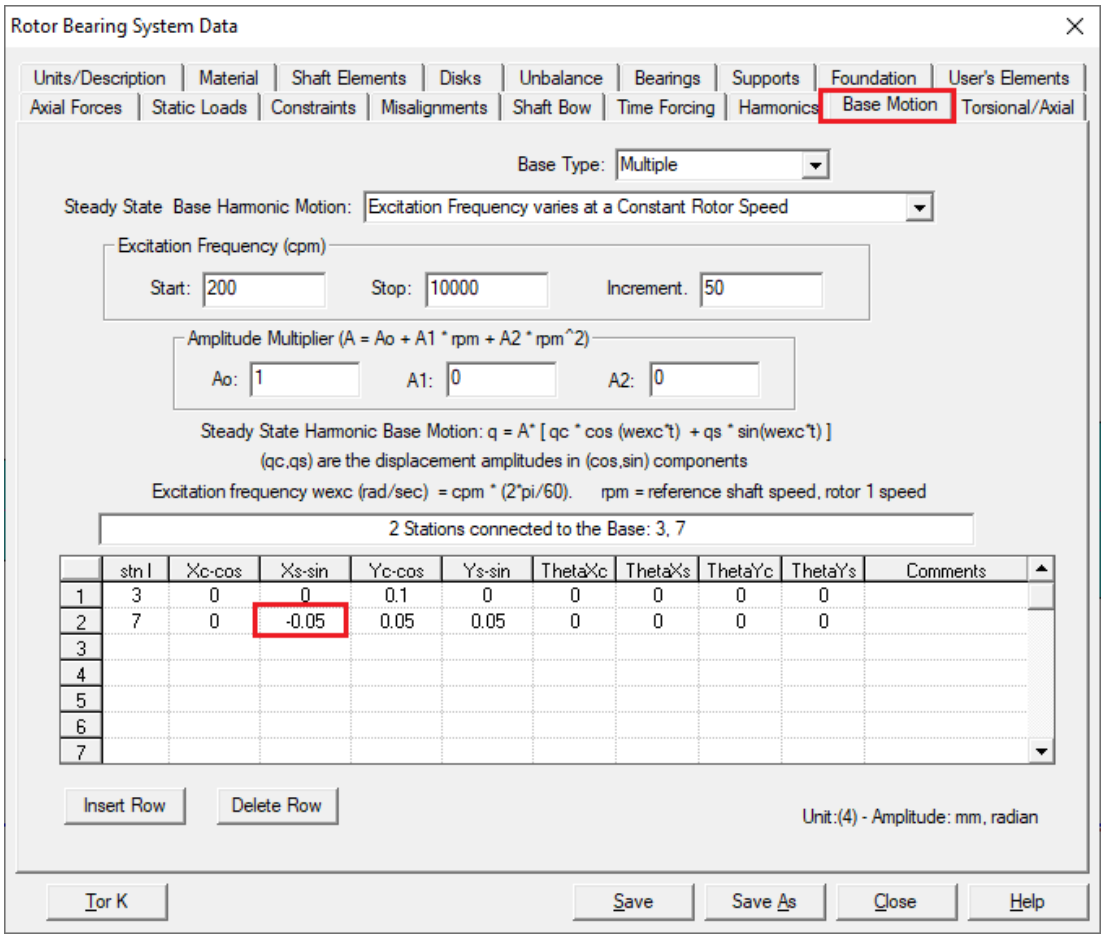

Elliptical Orbital Axes Plot
Options
Station
Elliptical Orbital Axes
Station: 1, Sub-Station: 1
Peak disp: max amp = 0.94843 at 4350 rpm
Negative (b) indicates backward precession
Semi-Minor (b)
0.05
0.00
-0.05
-0.10
-0.15
-0.20
Forward
Backward
Semi-Major (a)
1.10
0.88
0.66
0.44
0.22
0.00
0
2000
4000
6000
8000
10000
Excitation Frequency (cpm)


Elliptical Orbital Axes Plot
Options
Station
Elliptical Orbital Axes
Station: 7, Sub-Station: 1
Peak disp: max amp = 0.37239 at 7400 rpm
Negative (b) indicates backward precession
Semi-Minor (b)
0.03
0.00
-0.03
-0.06
-0.09
-0.12
-0.15
Semi-Major (a)
0.50
0.40
0.30
0.20
0.10
0.00
0
2000
4000
6000
8000
10000
Excitation Frequency (cpm)

Case 4: BaseMotion_3d.rot

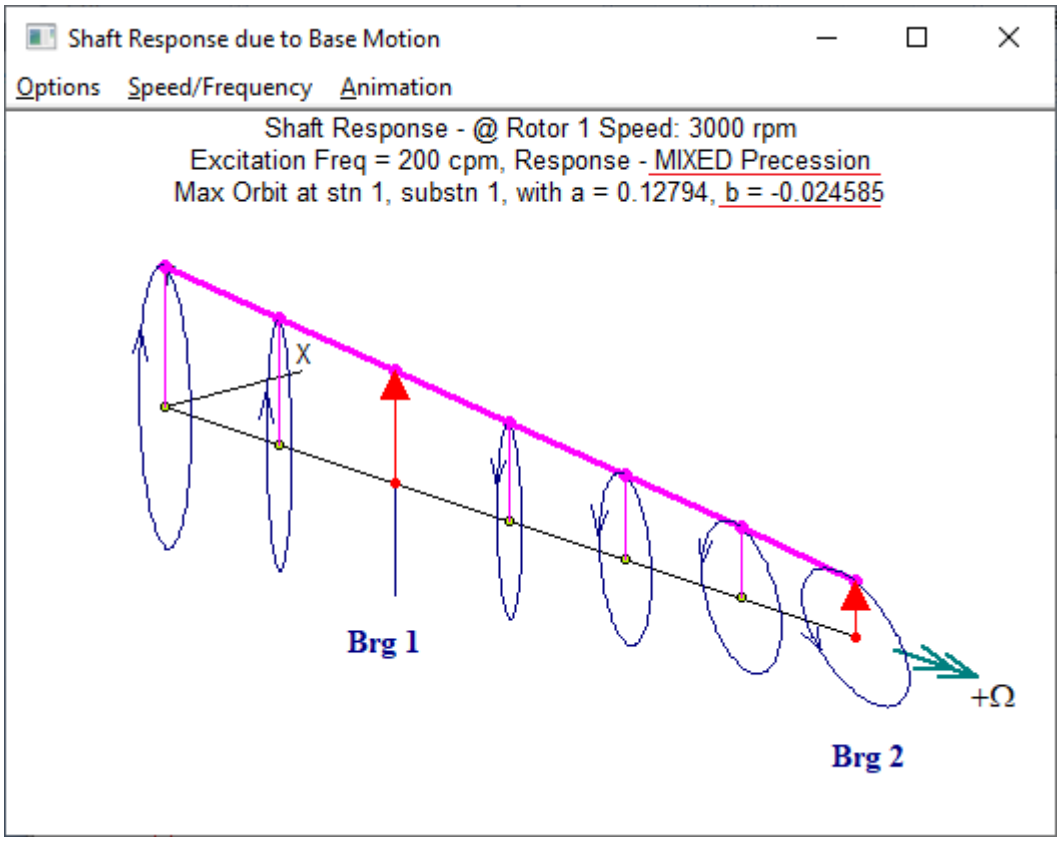
In this case, change the base motion x_s from 0.05 to -0.05 at station 7. This will change the base motion at station 7 from the previous backward precession to forward precession. The rests of data are unchanged.

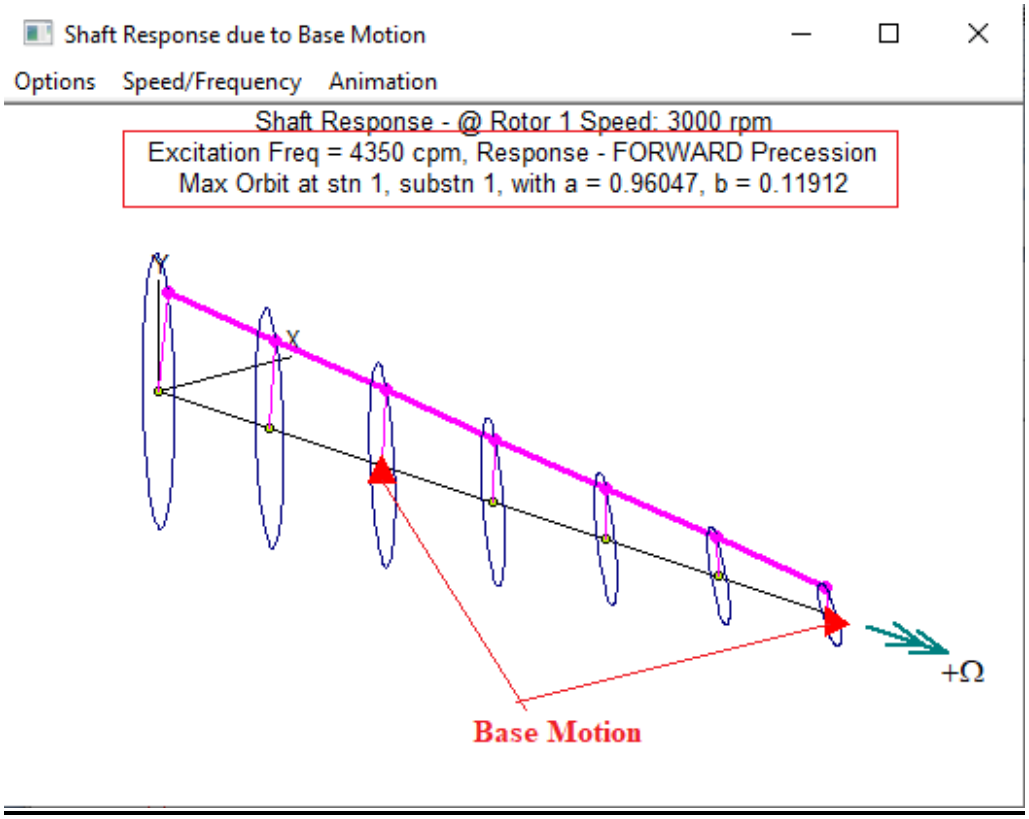


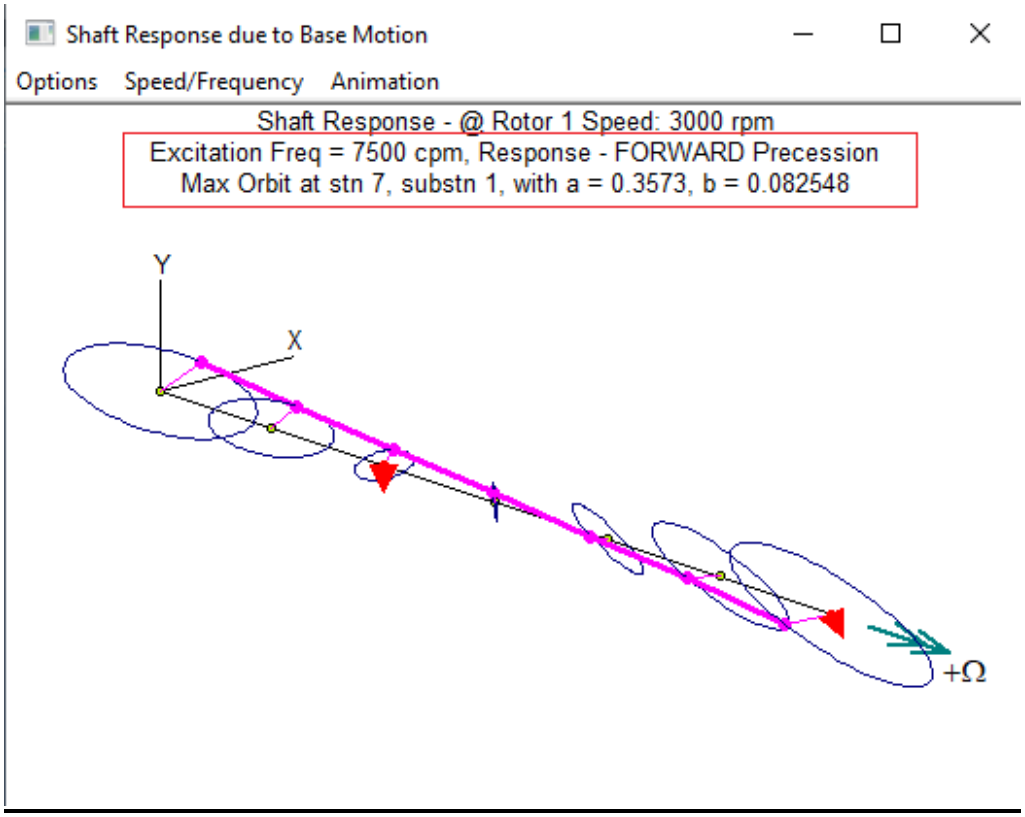


Now, at 200 cpm, again, at such low excitation frequency and low frequency ratio, the rotor moves with the base. It is still a mixed precession, however, the stations 1 and 2 before the bearing #1 (station 3) are backward precessions now, and after the straight line motion at station 3, the rotor whirls in the forward precession same as the station 7 base motion.

At 4350 cpm, the response is at and near resonance. The entire rotor whirls in the forward precession and the rotor response is far larger than the base motion. The rotor deflection shape is similar to the second mode. At 7500 cpm, the rotor whirls in the forward precession. The rotor deflection shape is similar to the fourth mode.

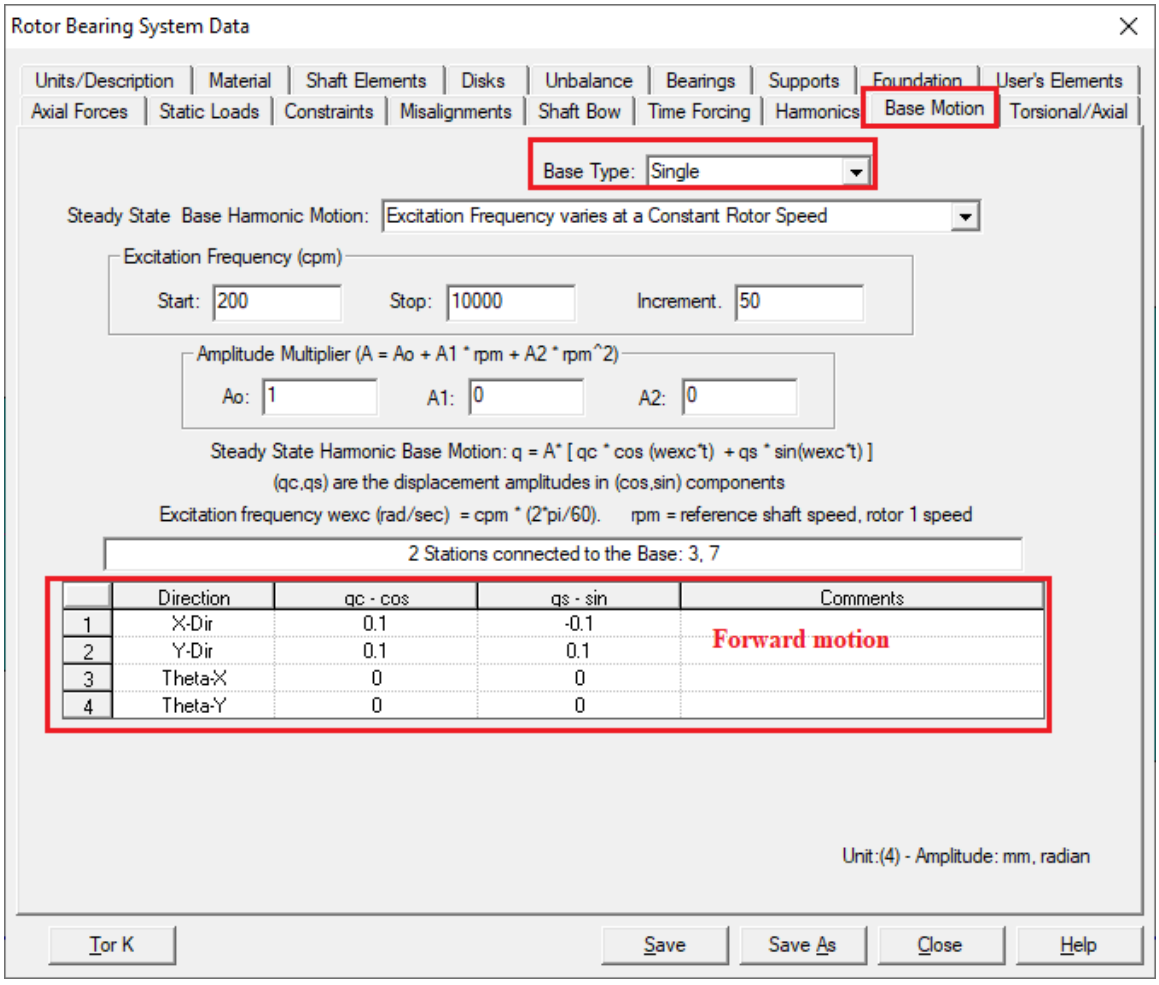


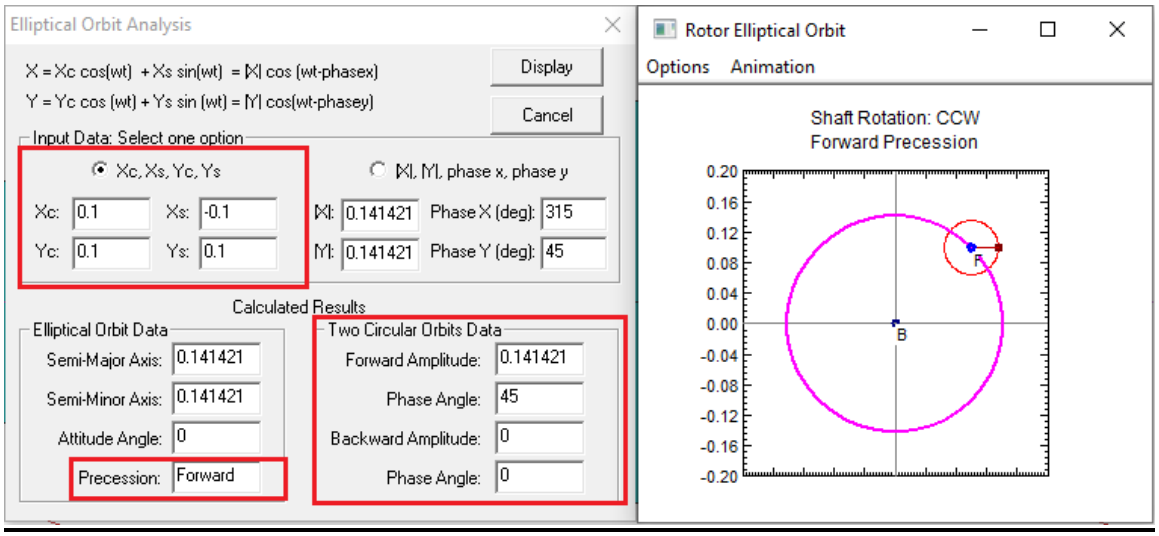

Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 4350 cpm, Response - FORWARD Precession
Max Orbit at stn 1, substn 1, with a = 0.96047, b = 0.11912
Y
X
+Ω
Base Motion


Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 7500 cpm, Response - FORWARD Precession
Max Orbit at stn 7, substn 1, with a = 0.3573, b = 0.082548
Y
X
+Ω

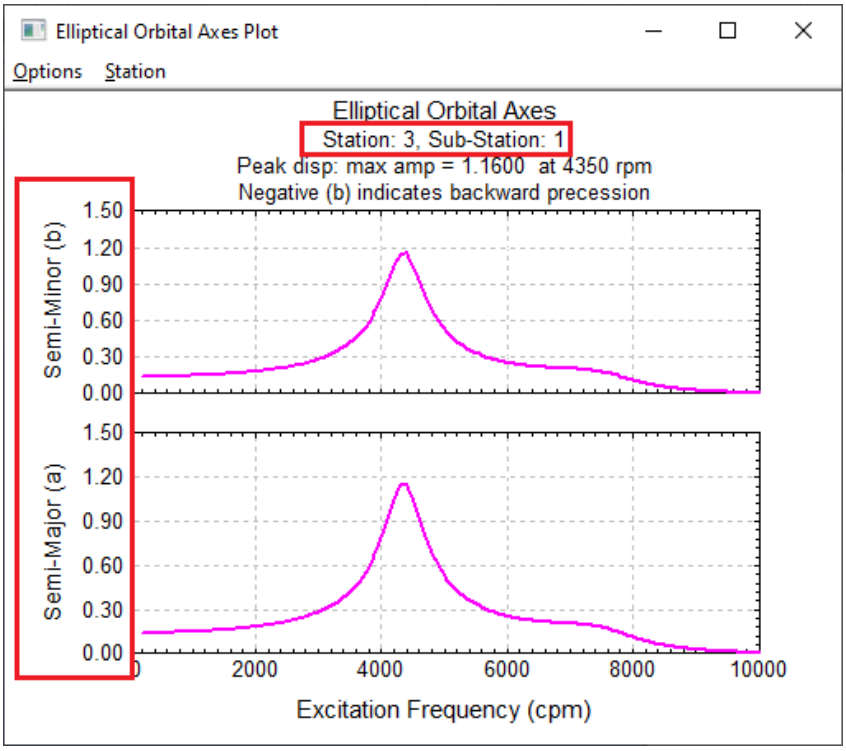
Case 5: BaseMotion_3e.rot

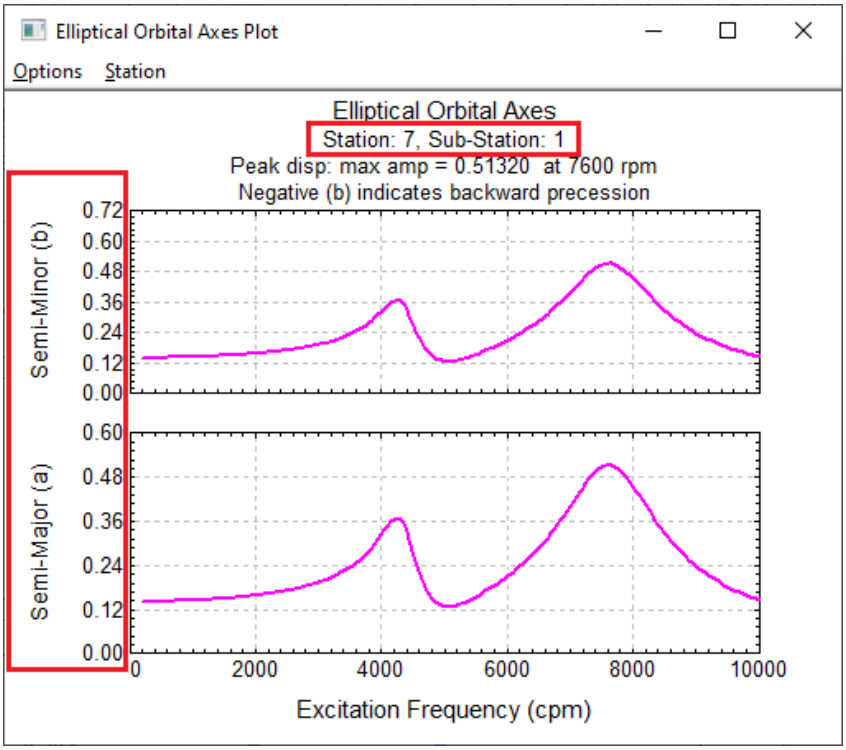
Let us consider the base motion is a purely forward circular motion as shown below:

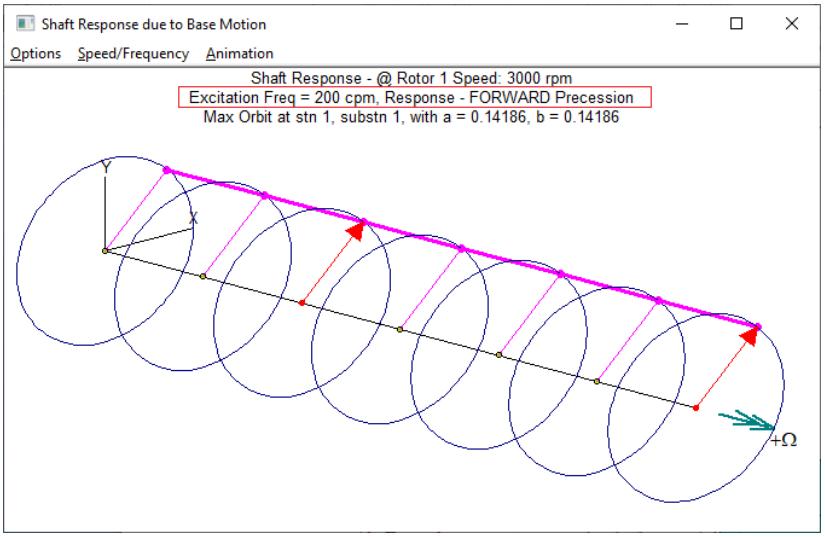


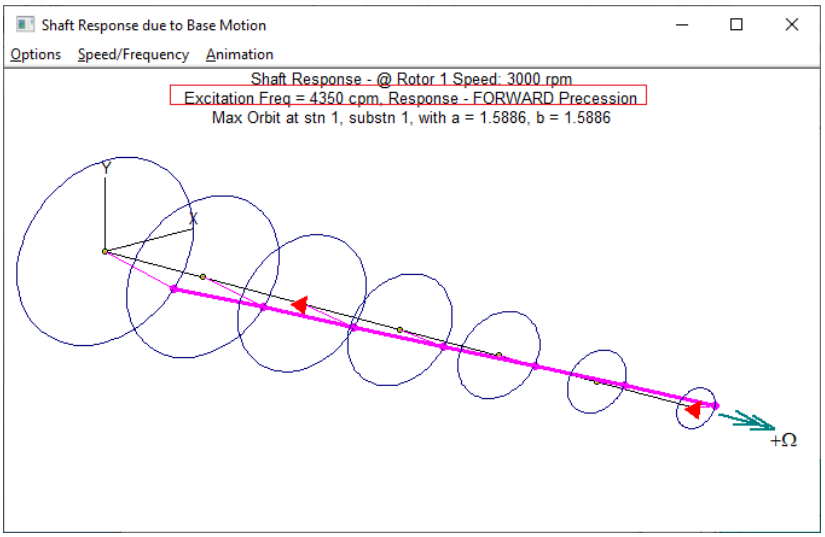


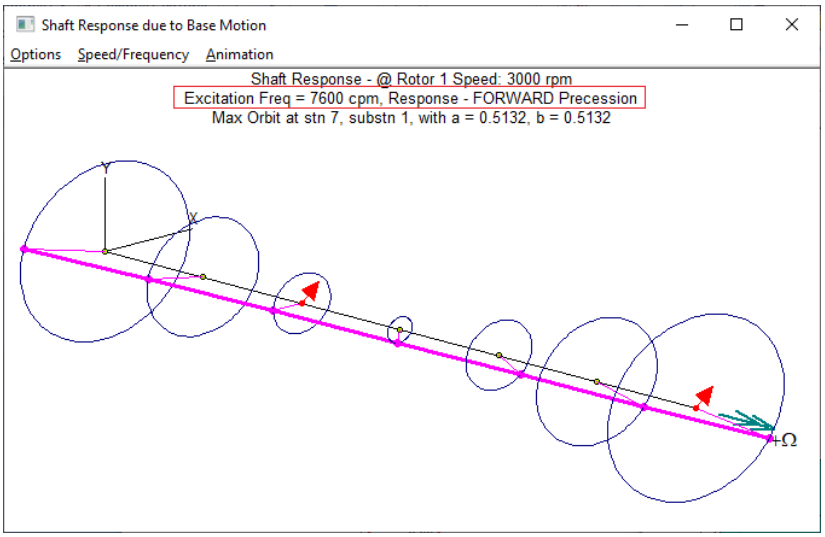
Now, you already know the results. Yes, this base motion will only excite the forward modes and no backward precession will be present. Since the system is an isotropic system, the rotor response orbits will be purely forward circular orbits.





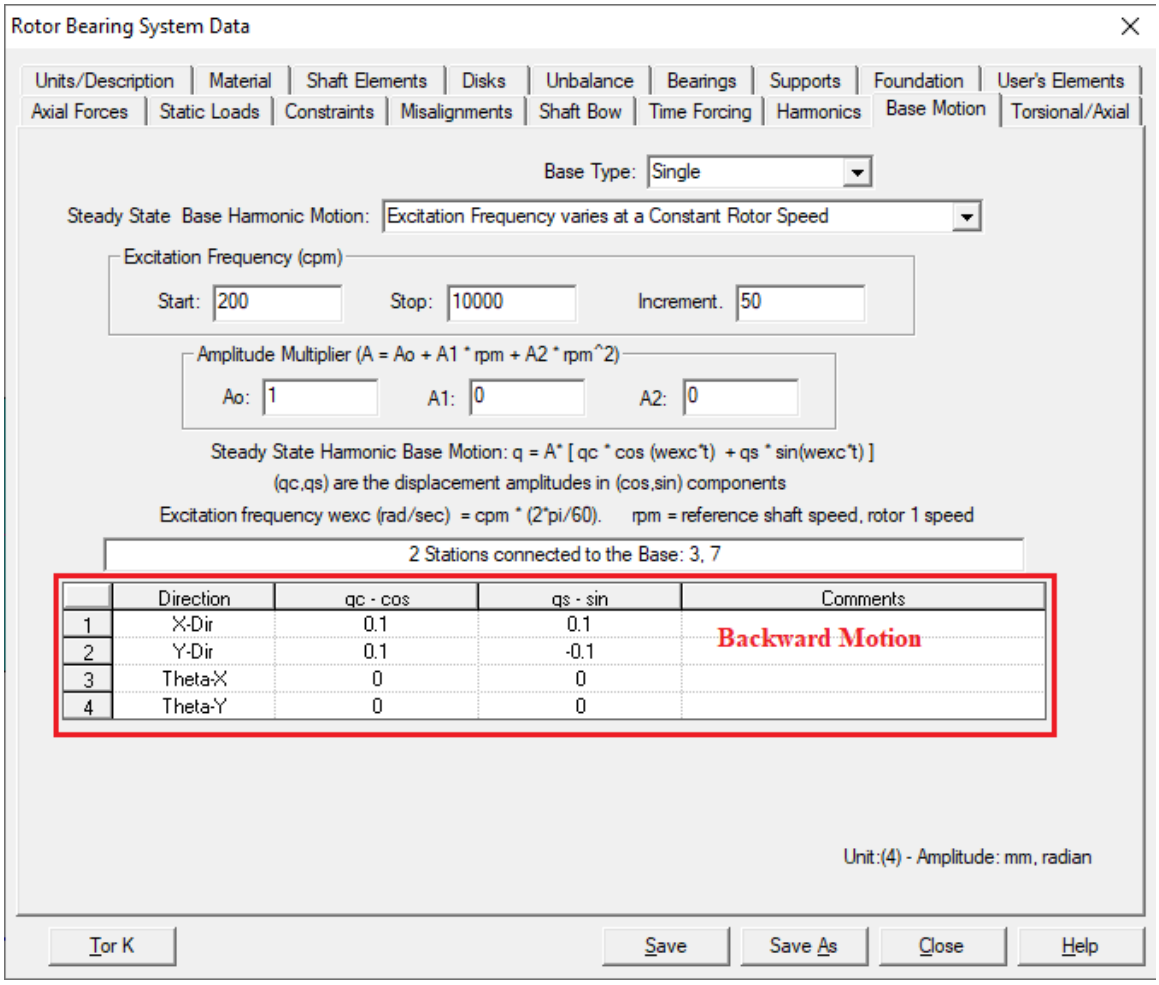

Shaft Response due to Base Motion
Options Speed/Frequency Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 200 cpm, Response - FORWARD Precession
Max Orbit at stn 1, substn 1, with a = 0.14186, b = 0.14186
Y
X
+Ω

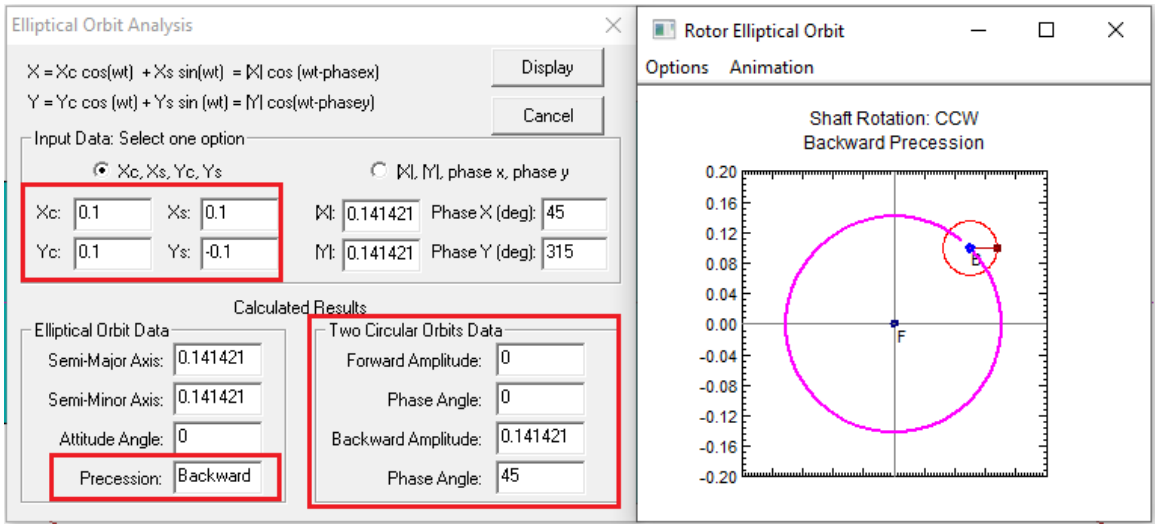

Shaft Response due to Base Motion
Options Speed/Frequency Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 4350 cpm, Response - FORWARD Precession
Max Orbit at stn 1, substn 1, with a = 1.5886, b = 1.5886
Y
X
+Ω


Shaft Response due to Base Motion
Options Speed/Frequency Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 7600 cpm, Response - FORWARD Precession
Max Orbit at stn 7, substn 1, with a = 0.5132, b = 0.5132
Y
X
+Ω

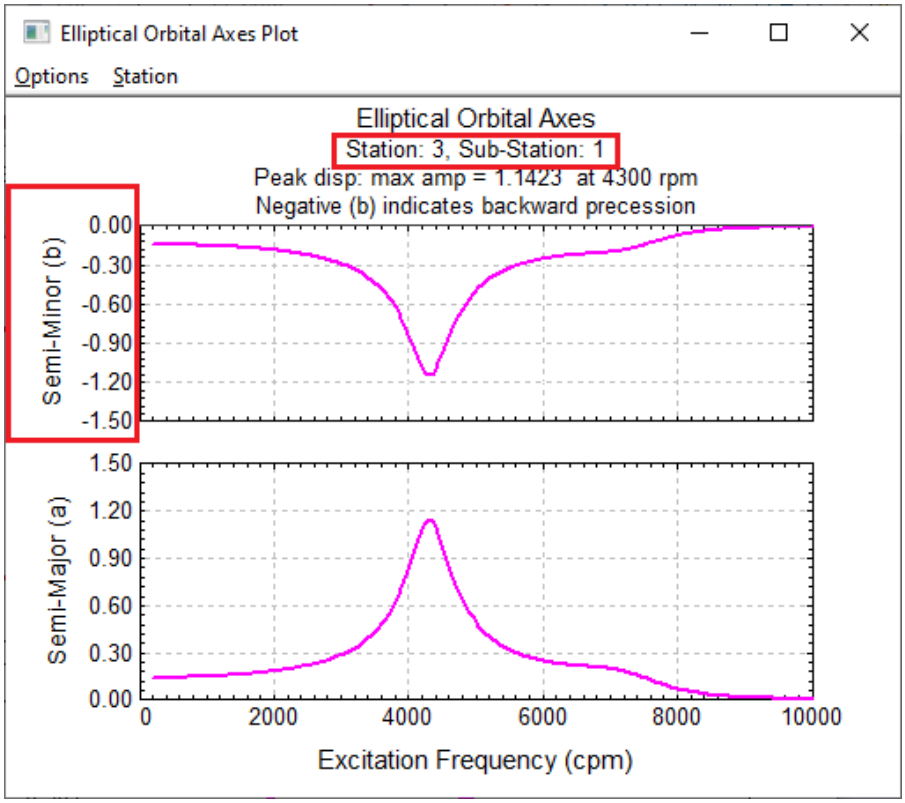
Case 6: BaseMotion_3f.rot

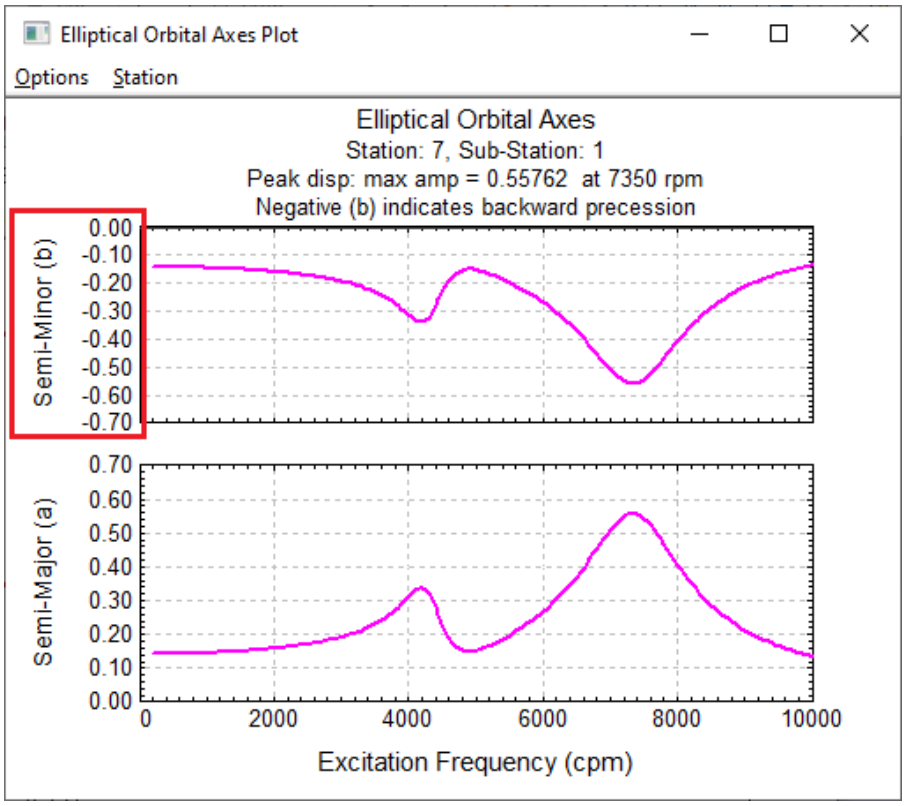
Consider a purely backward circular base motion in this example.

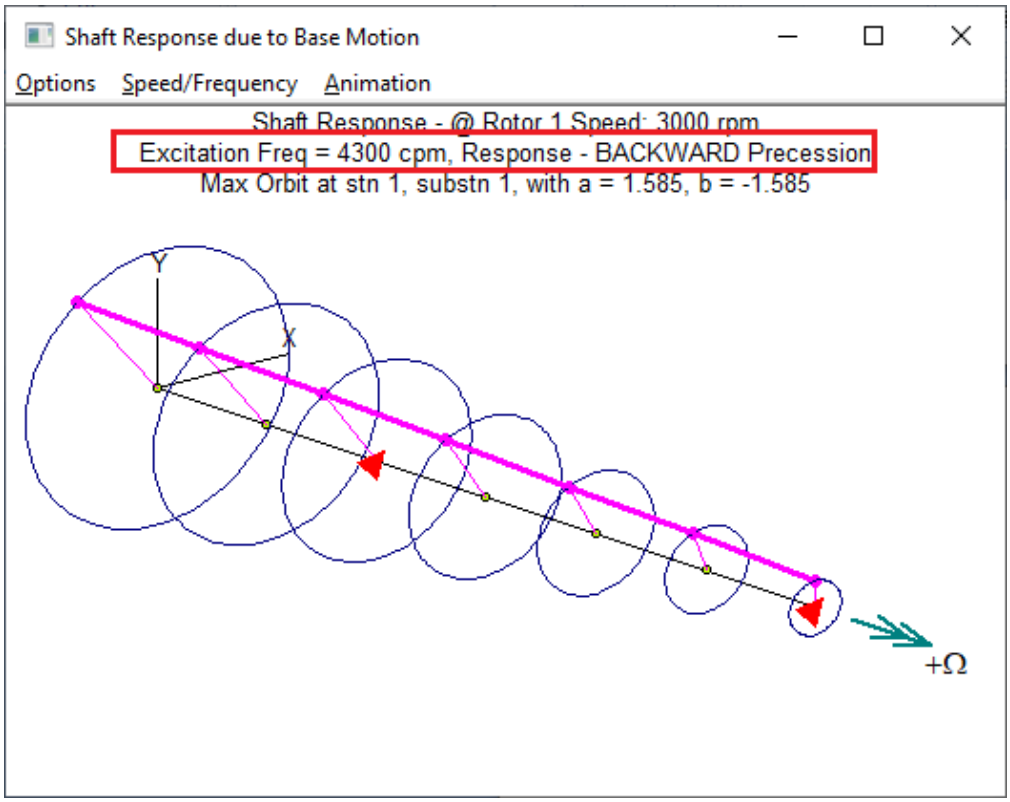


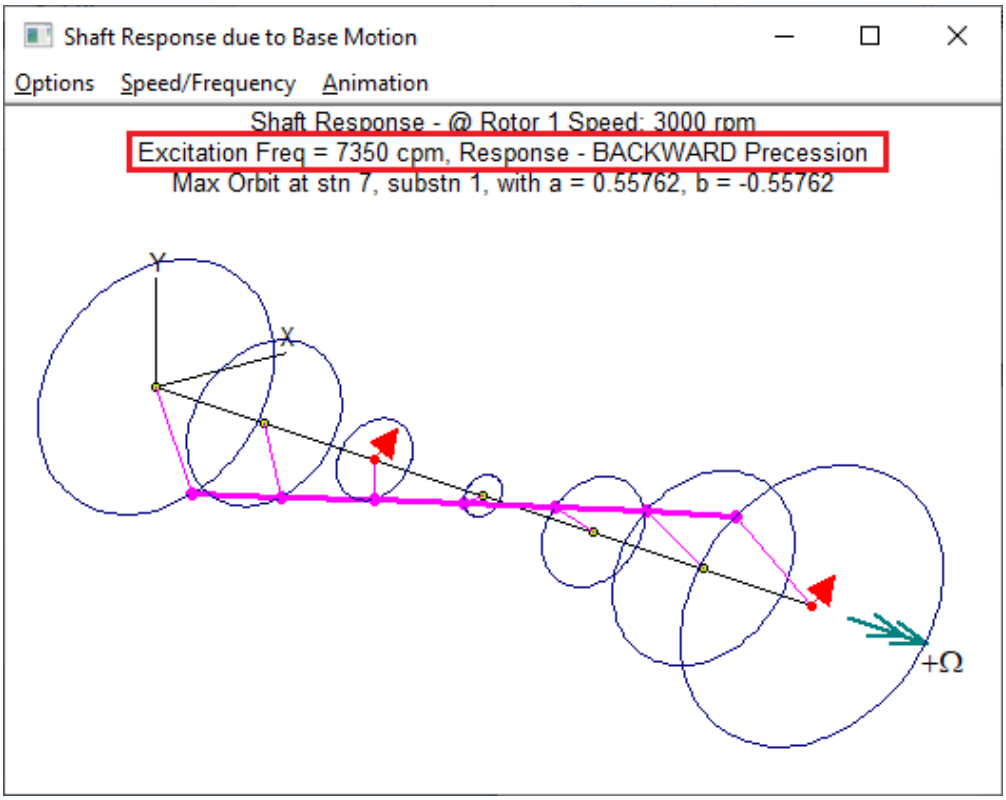


Results are as expected.






Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 4300 cpm, Response - BACKWARD Precession
Max Orbit at stn 1, substn 1, with a = 1.585, b = -1.585
Y
X
+Ω


Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 3000 rpm
Excitation Freq = 7350 cpm, Response - BACKWARD Precession
Max Orbit at stn 7, substn 1, with a = 0.55762, b = -0.55762
Y
X
+Ω

Example 4 – Industrial Compressor (BaseMotion_4a.rot)

An industrial compressor, as shown below, is used in this example. The rotor assembly is supported by two bearings at stations 2 and 4. The compressor design speed is 35,000 rpm. The bearing #1 at station 2 is a 3-lobe bearing and the bearing #2 at station 4 is a tilting pad bearing. Bearing Type 15 is used in this example.

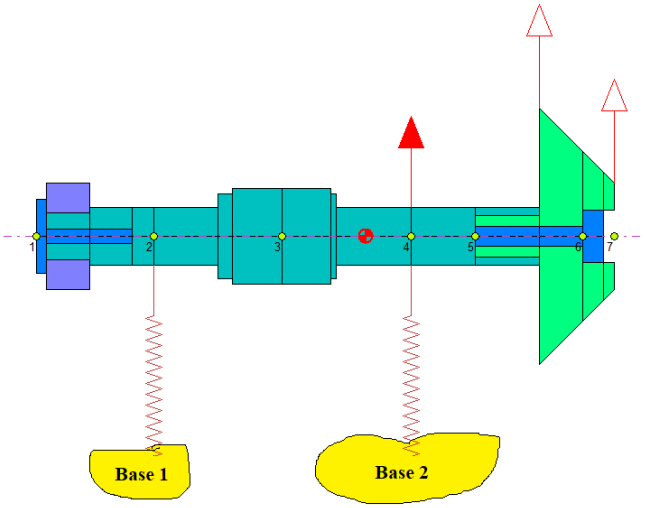
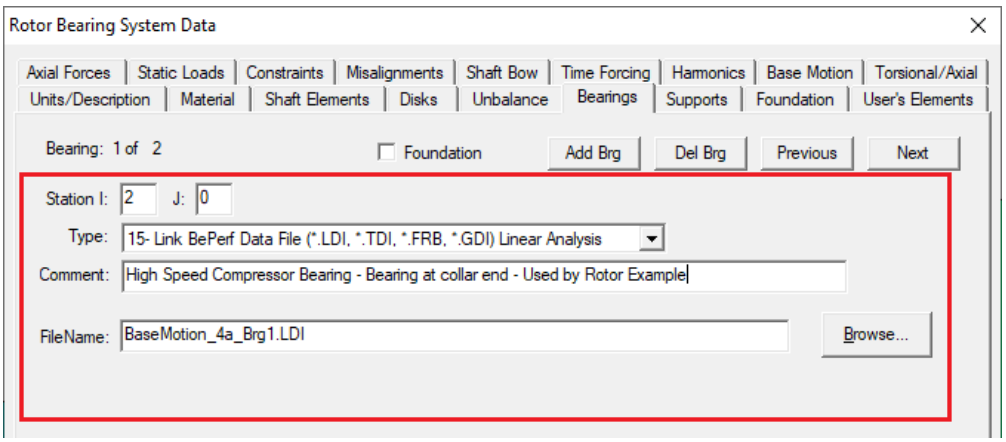
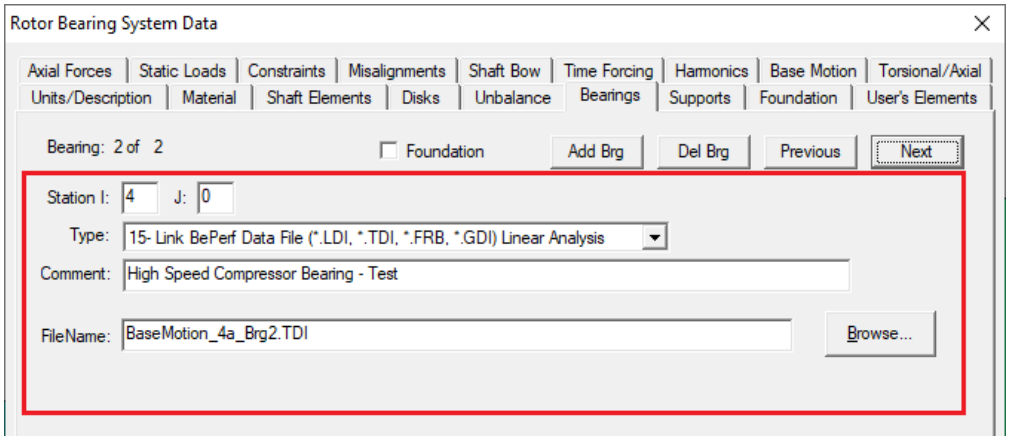


Fig. 35 – An Industrial Compressor





At 35,000 rpm, the first 4 precessional modes are shown in Fig. 36. It shows that the base motion will most likely excite the 1st and 2nd modes.

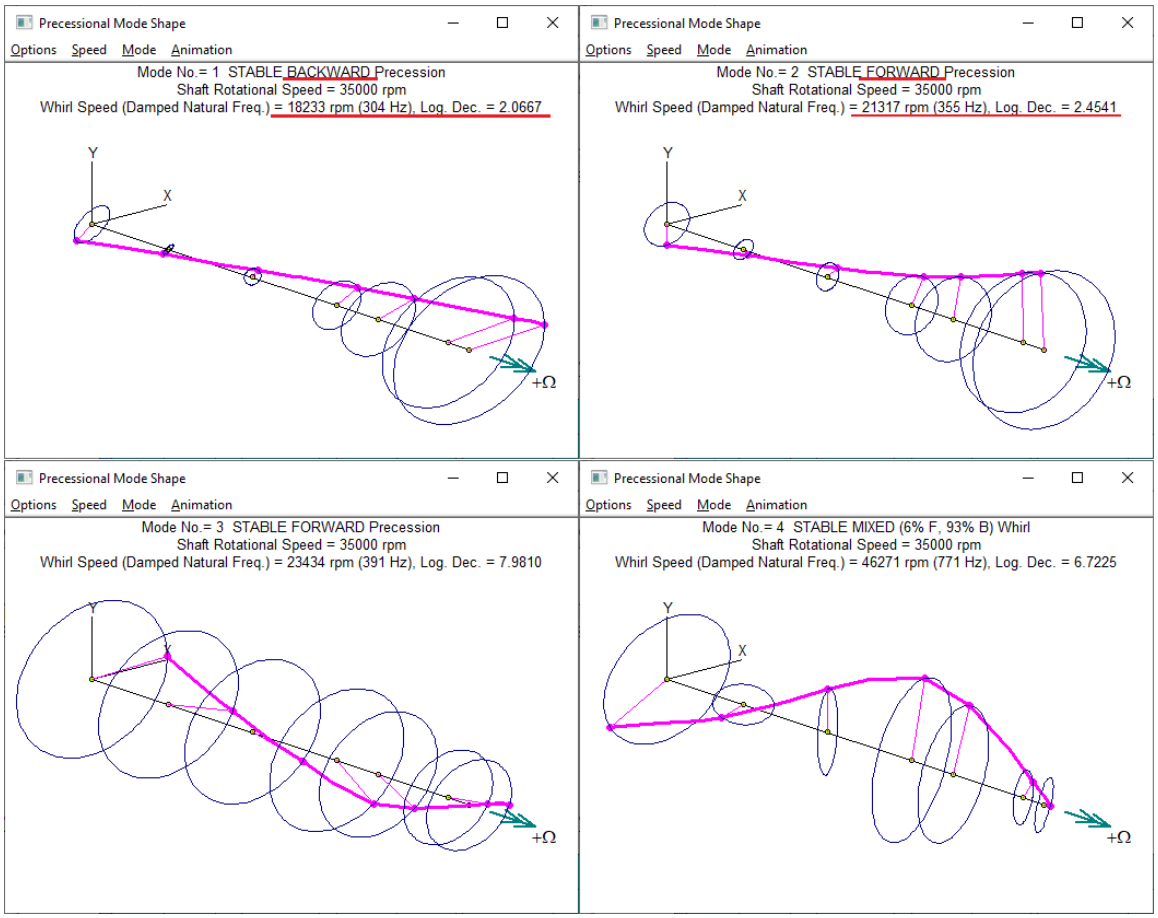


Fig. 36 – The first 4 Precessional Modes

Case 1: BaseMotion_4a.rot

For verification purposes, only the 2nd bearing at station 4 is subject to a base motion and the 1st bearing at station 2 is not subject to any base motion. Therefore, two bases are utilized in this model. The base 1 is not moving and the base 2 has a harmonic motion in Y direction. The base motion is shown in Fig. 37. The base excitation only at station 4 (base 2) varies from 1000 cpm to 50,000 cpm with an increment of 500 cpm at a constant rotor speed of 35,000 rpm.

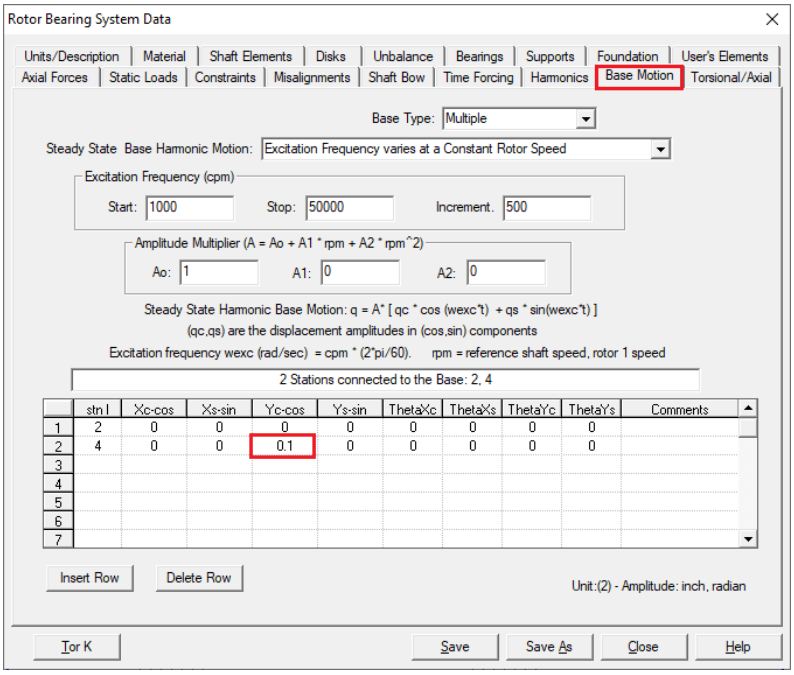
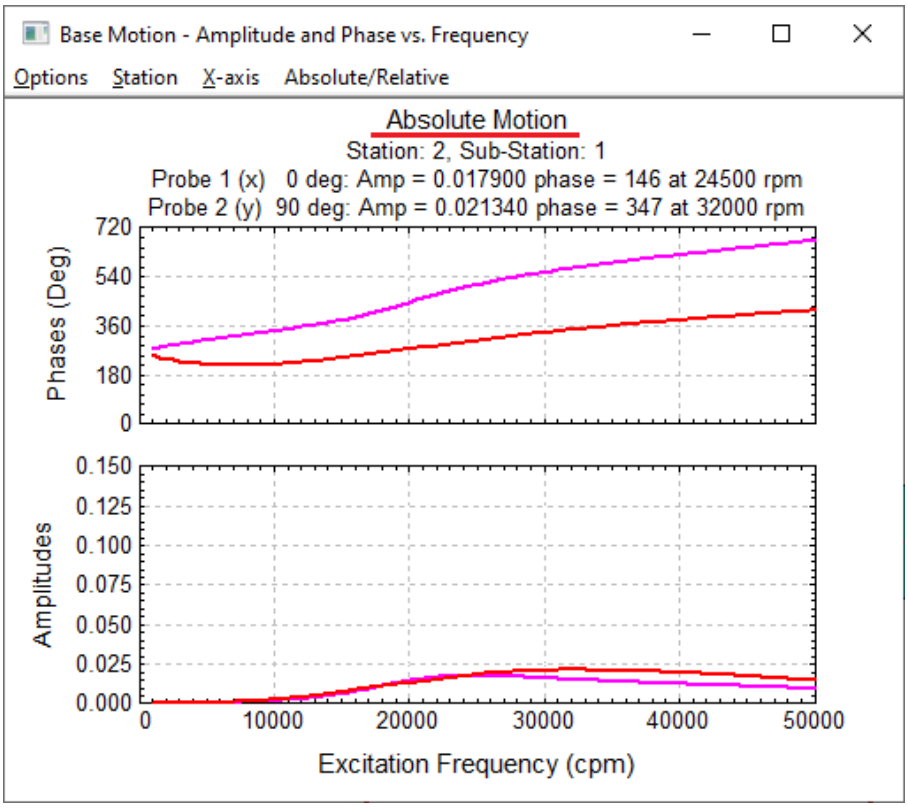


Fig. 37 – Base Motion

The absolute displacements at both bearings are shown in Fig. 38. The relative displacements are shown in Fig. 39. The bearing transmitted forces are shown in Fig. 40.



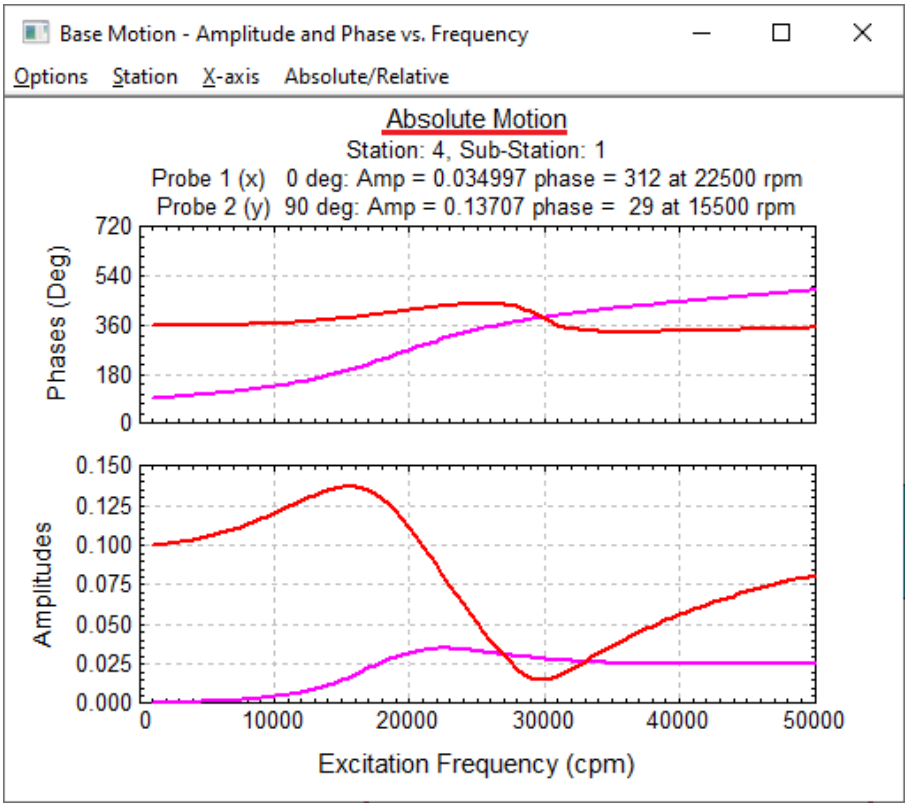
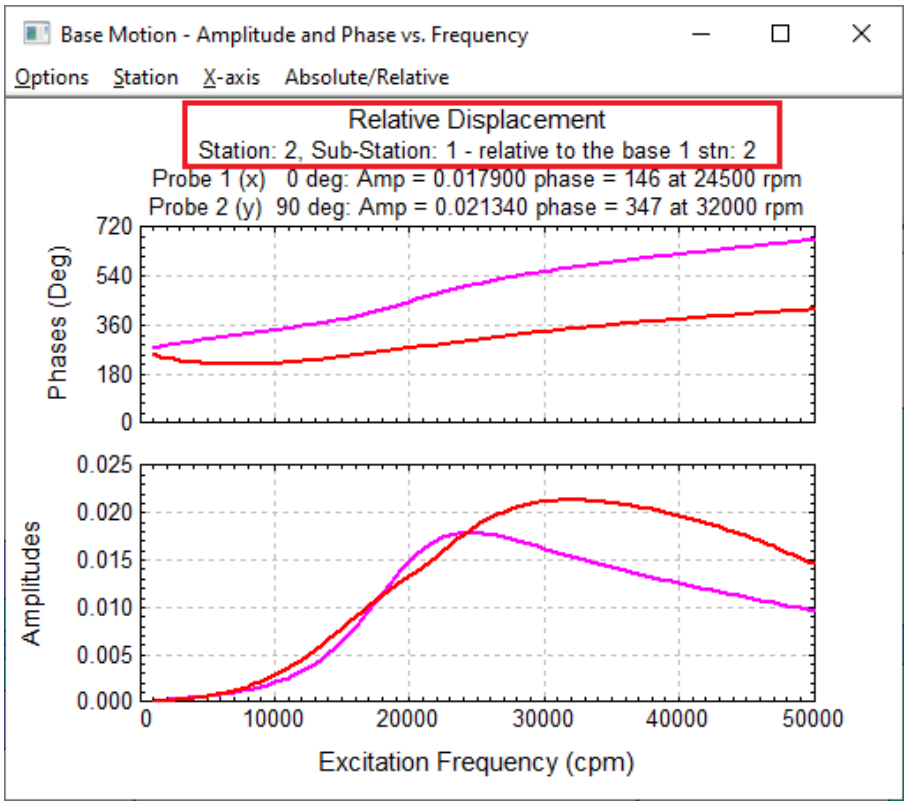


Fig. 38 – The Absolute Displacements due to Base Motion



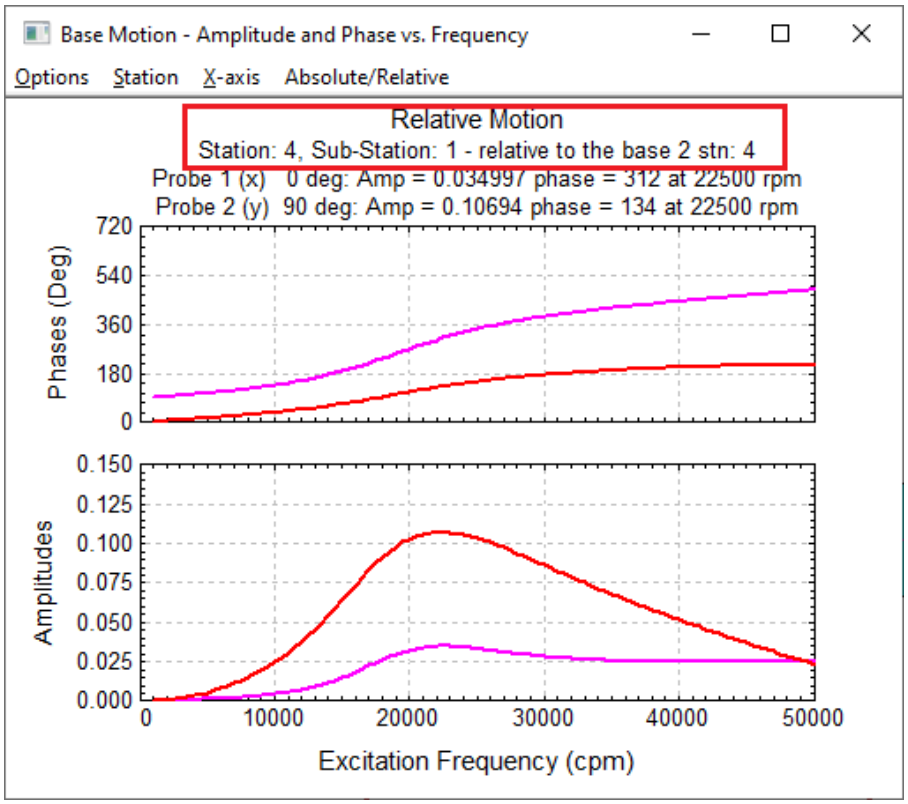
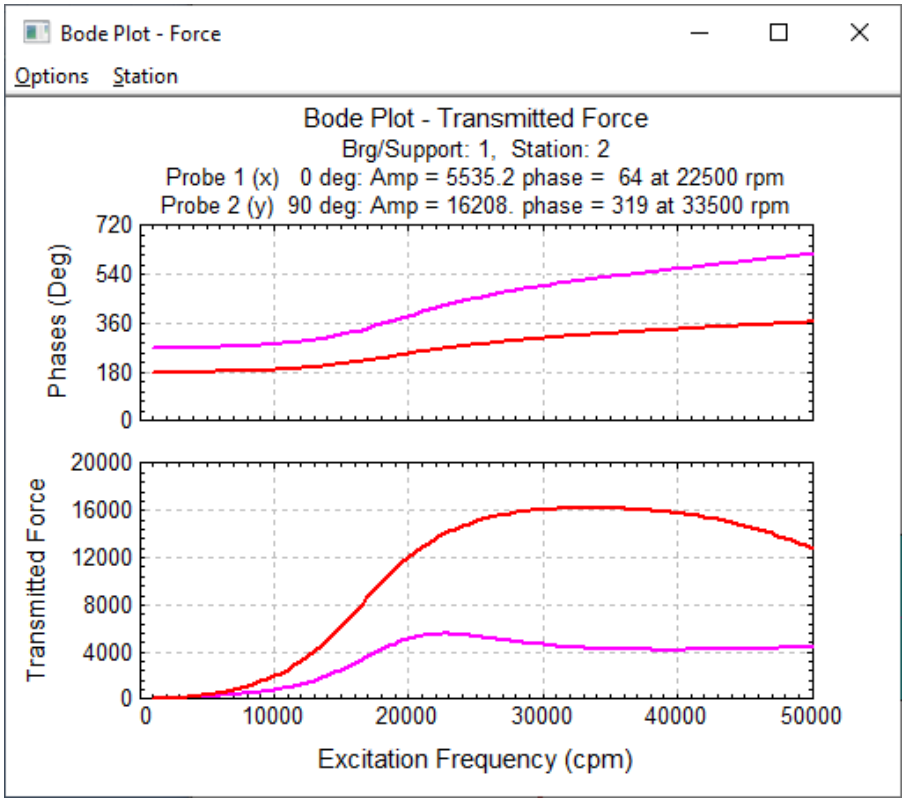


Fig. 39 – The Relative Displacement due to Base Motion



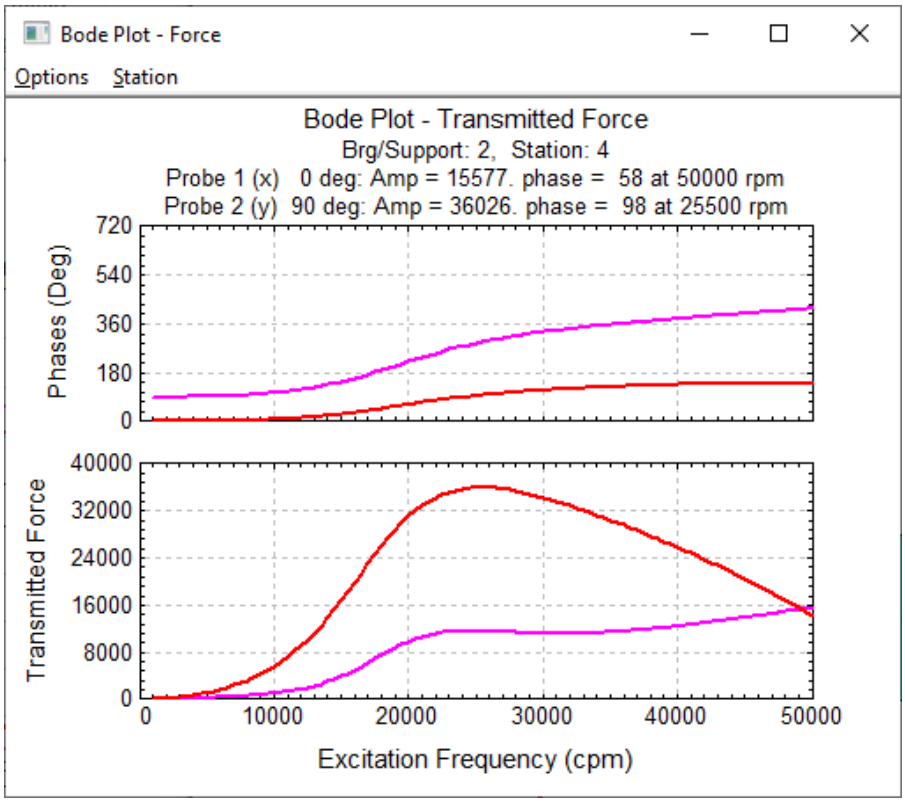
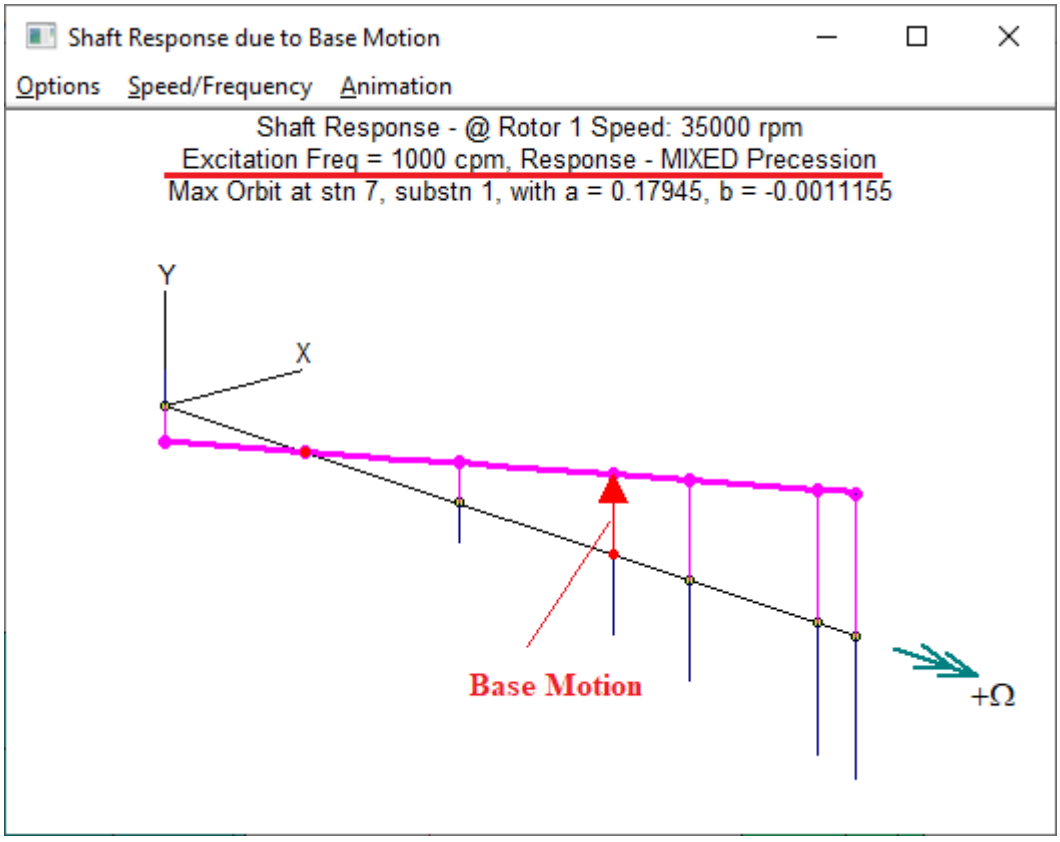


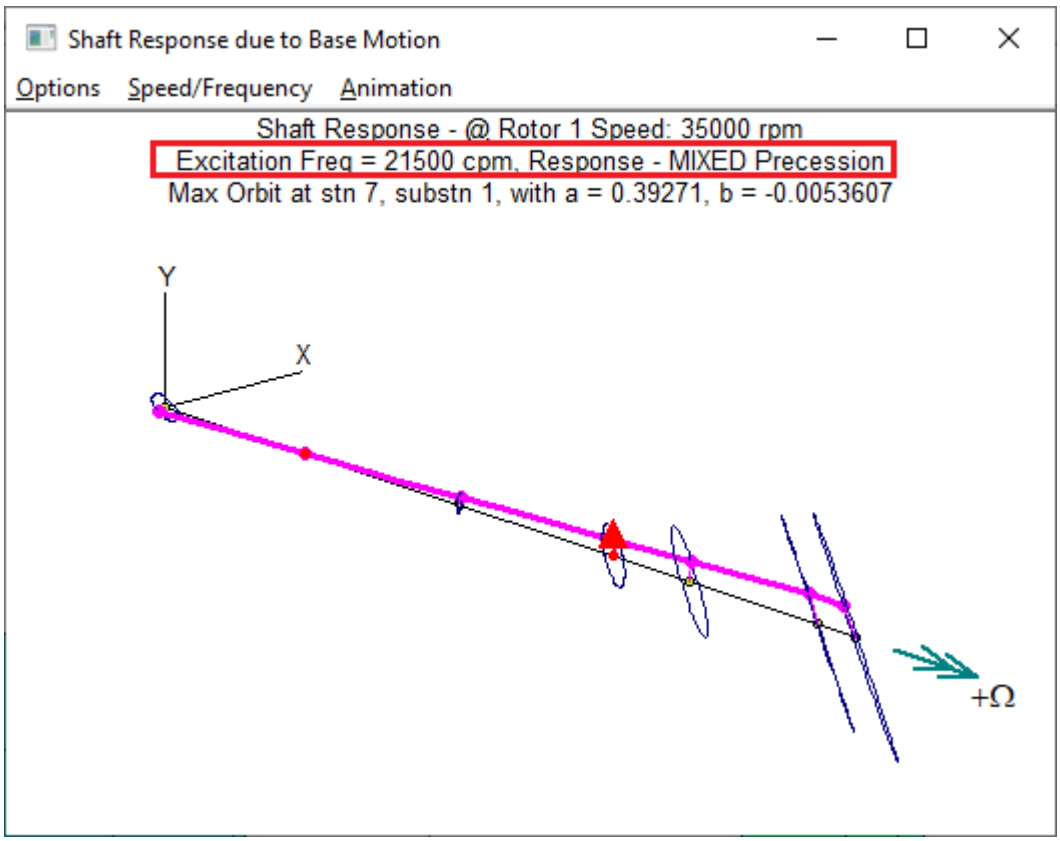
Fig. 40 – The Transmitted Bearing Forces due to Base Motion

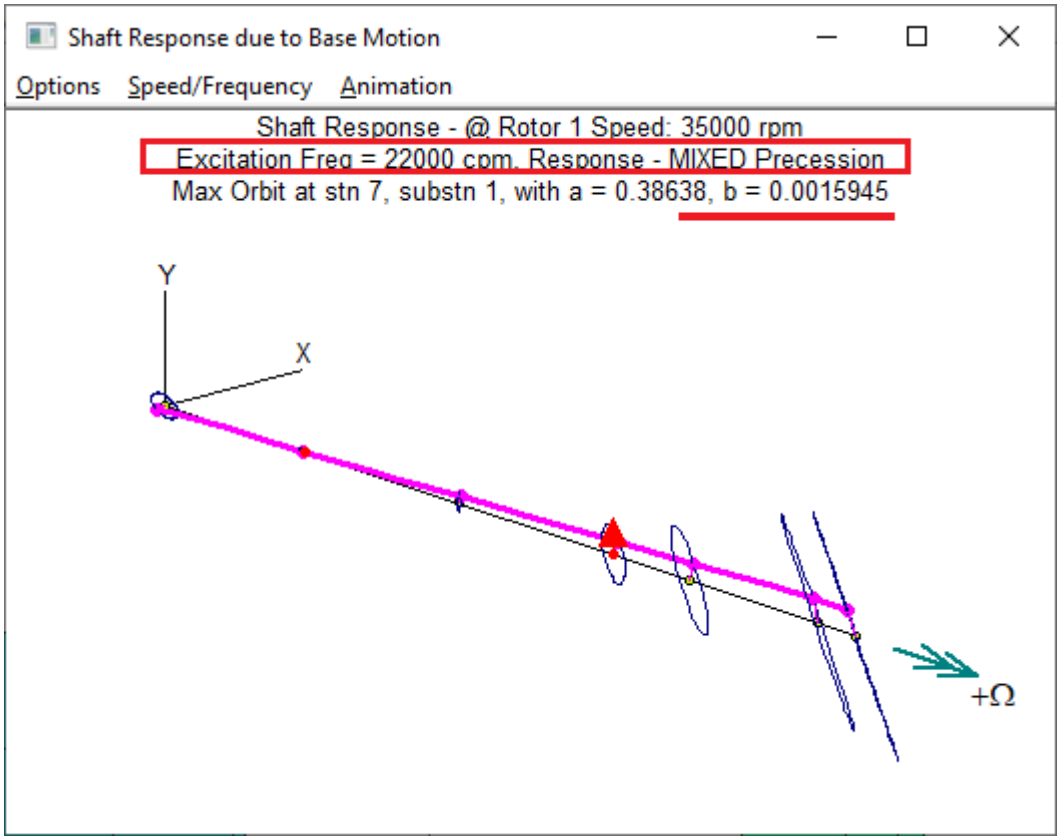
At low excitation frequency, 1000 cpm, again, the rotor moves with the base motion and there is little relative movement between the rotor and base. Also, the straight line base motion can excite both forward and backward precessions. In this case and at this low frequency, the rotor moves nearly straight line motion (b=0) which is the same as the base motion.

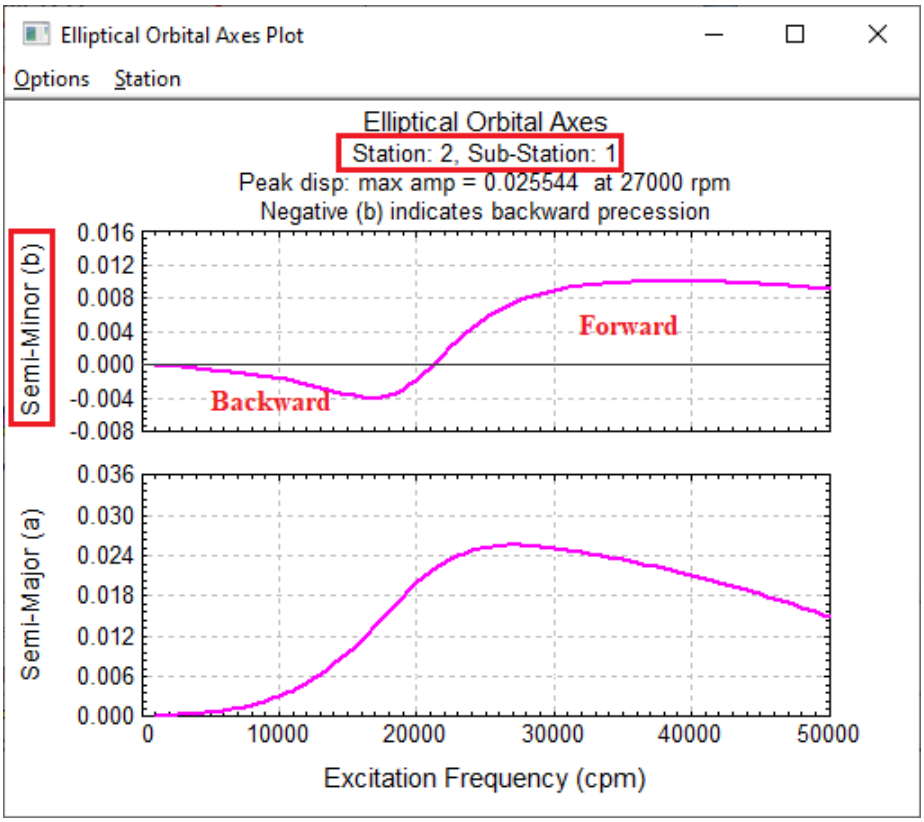
The maximum response occurs at the impeller side (station 7). Again, at low excitation frequency, the rotor moves with the base motion (straight line) and proceeds with a backward whirl, then reaches the resonance at 19,500 cpm. The motion becomes forward precession after 21,500 cpm, This is understandable since the 1st mode at 18,233 cpm with a log. Decrement of 2.0667 is a backward mode and the 2nd mode at 21,317 cpm with a log. Decrement of 2.4541 is a forward mode.

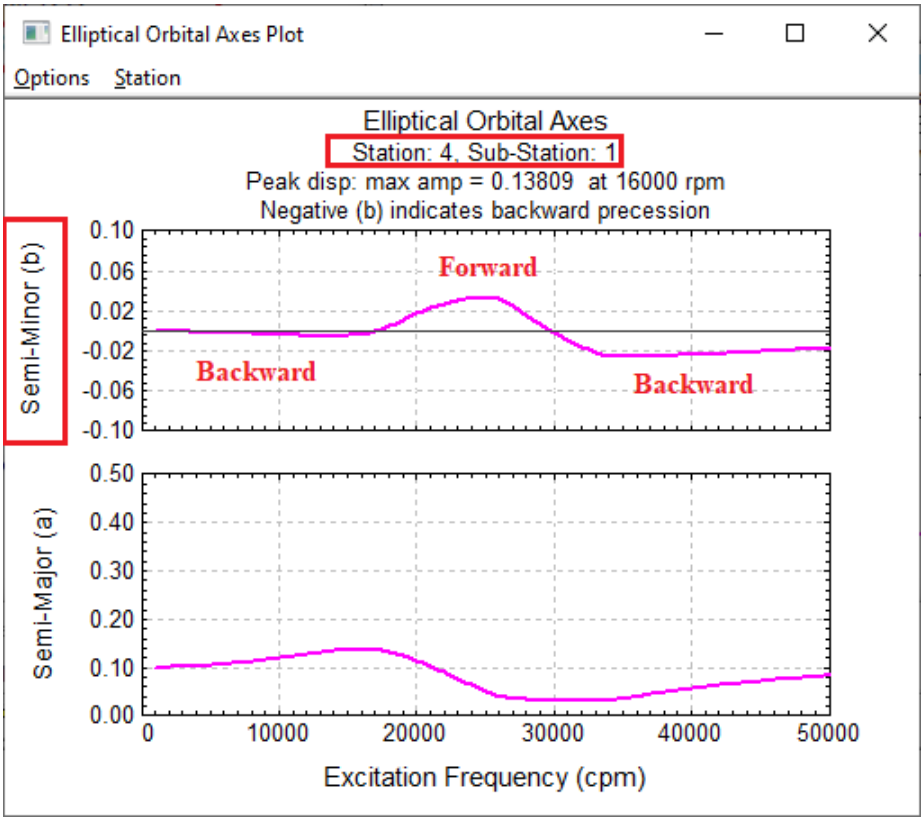
At station 4 where the base motion occurs, the motion starts from a straight line motion, then backward, and forward, and backward.

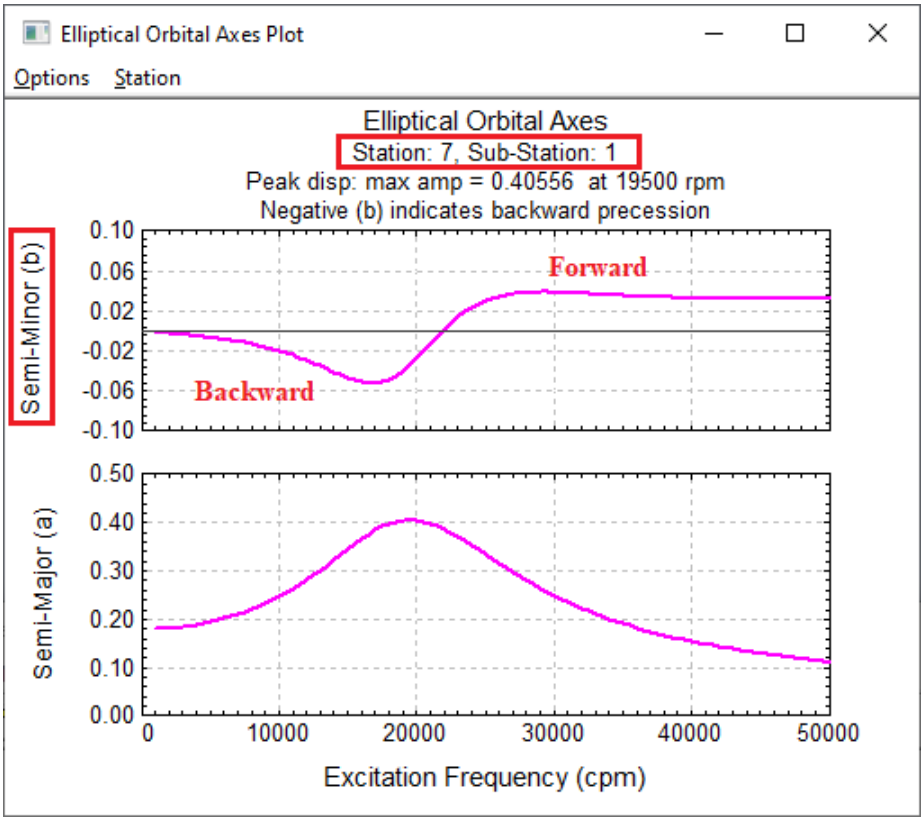

Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 35000 rpm
Excitation Freq = 1000 cpm, Response - MIXED Precession
Max Orbit at stn 7, substn 1, with a = 0.17945, b = -0.0011155
Y
X
Base Motion
+Ω


Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 35000 rpm
Excitation Freq = 21500 cpm, Response - MIXED Precession
Max Orbit at stn 7, substn 1, with a = 0.39271, b = -0.0053607
Y
X
+Ω


Shaft Response due to Base Motion
Options
Speed/Frequency
Animation
Shaft Response - @ Rotor 1 Speed: 35000 rpm
Excitation Freq = 22000 cpm, Response - MIXED Precession
Max Orbit at stn 7, substn 1, with a = 0.38638, b = 0.0015945
Y
X
+Ω


Elliptical Orbital Axes Plot
Options
Station
Elliptical Orbital Axes
Station: 2, Sub-Station: 1
Peak disp: max amp = 0.025544 at 27000 rpm
Negative (b) indicates backward precession
Semi-Minor (b)
0.016
0.012
0.008
0.004
0.000
-0.004
-0.008
Forward
Backward
Semi-Major (a)
0.036
0.030
0.024
0.018
0.012
0.006
0.000
0
10000
20000
30000
40000
50000
Excitation Frequency (cpm)


Elliptical Orbital Axes Plot
Options
Station
Elliptical Orbital Axes
Station: 4, Sub-Station: 1
Peak disp: max amp = 0.13809 at 16000 rpm
Negative (b) indicates backward precession
Semi-Minor (b)
0.10
0.06
0.02
-0.02
-0.06
-0.10
Forward
Backward
Backward
Semi-Major (a)
0.50
0.40
0.30
0.20
0.10
0.00
0
10000
20000
30000
40000
50000
Excitation Frequency (cpm)


Elliptical Orbital Axes Plot
Options
Station
Elliptical Orbital Axes
Station: 7, Sub-Station: 1
Peak disp: max amp = 0.40556 at 19500 rpm
Negative (b) indicates backward precession
Semi-Minor (b)
0.10
0.06
0.02
-0.02
-0.06
-0.10
Forward
Backward
Semi-Major (a)
0.50
0.40
0.30
0.20
0.10
0.00
0
10000
20000
30000
40000
50000
Excitation Frequency (cpm)

The bearing coefficients for the bearing 2 (station 4) at 35,000 rpm are calculated below:

```
Brg Coefficients
Kxx          Kxy          Kyx          Kyy          Cxx          Cxy          Cyx          Cyy
197434.   0.00000      0.00000      197434.      109.352      0.00000      0.00000      109.352
```

Based on Eq. (13), we can calculate the equivalent harmonic excitation at the frequency of 22,500 cpm as shown in Fig. 41.

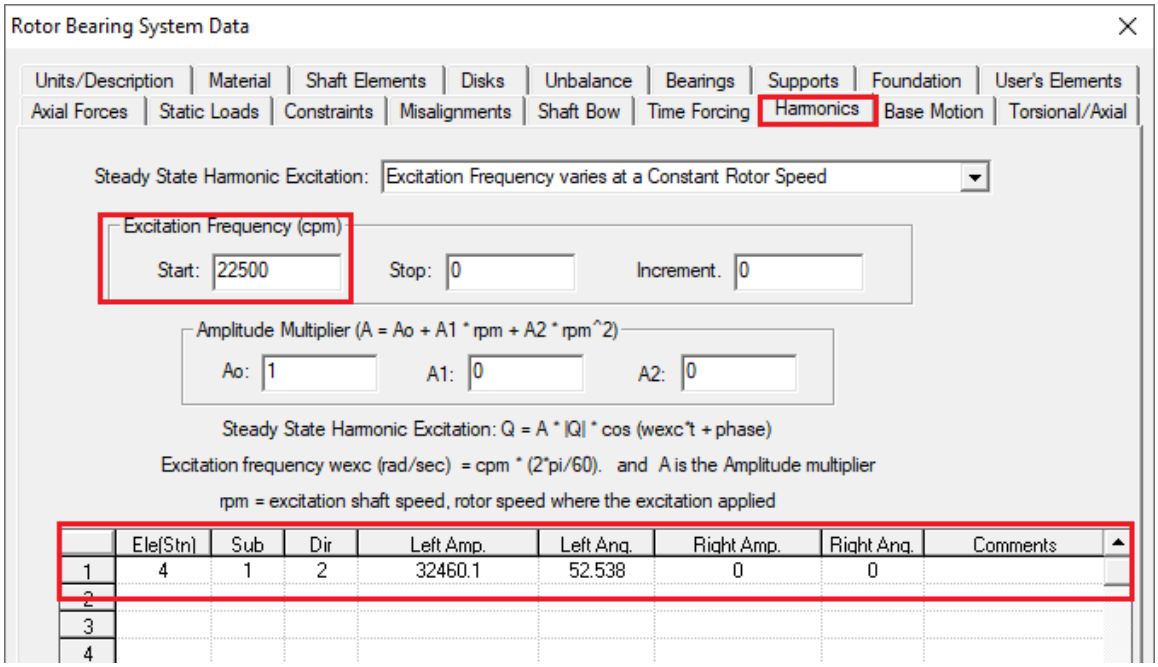


Fig. 41 – The Equivalent Harmonic Excitation at frequency of 22,500 cpm

The absolute displacements due to base motion and steady state harmonic excitation at the frequency of 22,500 cpm are identical as expected.

```
************** Harmonic Response due to Shaft (1) Excitation ***************
              *** Excitation Frequency =  22500.      cpm ***

  Shaft  1     Speed=   35000.00 rpm  =   3665.19 R/S  =     583.33 Hz

*********************** Shaft Element Displacements ************************
         =====  X  =====    =====  Y  =====        Elliptical Orbit Data
 stn sub Amplitude Phase    Amplitude Phase        A          B          G

  1   1  0.419E-01  134.7   0.421E-01  267.1    0.543E-01   0.240E-01   135.
      2  0.396E-01  134.2   0.396E-01  267.9    0.515E-01   0.220E-01   135.
      3  0.306E-01  131.4   0.297E-01  272.8    0.402E-01   0.141E-01   136.
      4  0.217E-01  126.2   0.203E-01  282.2    0.290E-01   0.615E-02   137.
  2   1  0.174E-01  121.7   0.160E-01  290.9    0.236E-01   0.221E-02   137.
      2  0.735E-02   78.6   0.101E-01  355.0    0.102E-01  -0.725E-02    80.
      3  0.638E-02   55.0   0.116E-01   15.9    0.127E-01  -0.366E-02    65.
  3   1  0.108E-01  351.9   0.227E-01   49.6    0.235E-01   0.879E-02    73.
      2  0.194E-01  333.5   0.375E-01   60.7    0.375E-01   0.193E-01    88.
```

```
      3  0.203E-01  332.4   0.392E-01   61.5    0.392E-01   0.203E-01    89.
  4   1  0.350E-01  311.9   0.812E-01   71.5    0.833E-01   0.297E-01   104.
  5   1  0.547E-01  290.9   0.153E+00   76.9    0.160E+00   0.293E-01   107.
      2  0.827E-01  275.5   0.247E+00   79.6    0.259E+00   0.216E-01   108.
  6   1  0.103E+00  268.8   0.311E+00   80.7    0.328E+00   0.139E-01   108.
      2  0.114E+00  266.3   0.344E+00   81.0    0.362E+00   0.998E-02   108.
  7   1  0.120E+00  265.3   0.360E+00   81.2    0.379E+00   0.804E-02   108.

*****************************************************************************
```

```
************ Harmonic Response due to Base Motion (Excitation) *************
              *** Excitation Frequency =  22500.      cpm ***

  Shaft  1     Speed=   35000.00 rpm  =   3665.19 R/S  =     583.33 Hz

********************** Shaft Element Displacements ************************
         =====  X  =====    =====  Y  =====        Elliptical Orbit Data
 stn sub Amplitude Phase    Amplitude Phase        A          B         G

  1   1  0.419E-01  134.7   0.421E-01  267.1    0.543E-01   0.240E-01   135.
      2  0.396E-01  134.2   0.396E-01  267.9    0.515E-01   0.220E-01   135.
      3  0.306E-01  131.4   0.297E-01  272.8    0.402E-01   0.141E-01   136.
      4  0.217E-01  126.2   0.203E-01  282.2    0.290E-01   0.615E-02   137.
  2   1  0.174E-01  121.7   0.160E-01  290.9    0.236E-01   0.221E-02   137.
      2  0.735E-02   78.6   0.101E-01  355.0    0.102E-01  -0.725E-02    80.
      3  0.638E-02   55.0   0.116E-01   15.9    0.127E-01  -0.366E-02    65.
  3   1  0.108E-01  351.9   0.227E-01   49.6    0.235E-01   0.879E-02    73.
      2  0.194E-01  333.5   0.375E-01   60.7    0.375E-01   0.193E-01    88.
      3  0.203E-01  332.4   0.392E-01   61.5    0.392E-01   0.203E-01    89.
  4   1  0.350E-01  311.9   0.812E-01   71.5    0.833E-01   0.297E-01   104.
  5   1  0.547E-01  290.9   0.153E+00   76.9    0.160E+00   0.293E-01   107.
      2  0.827E-01  275.5   0.247E+00   79.6    0.259E+00   0.216E-01   108.
  6   1  0.103E+00  268.8   0.311E+00   80.7    0.328E+00   0.139E-01   108.
      2  0.114E+00  266.3   0.344E+00   81.0    0.362E+00   0.998E-02   108.
  7   1  0.120E+00  265.3   0.360E+00   81.2    0.379E+00   0.804E-02   108.

*****************************************************************************
```

Case 2: BaseMotion_4b.rot

In this case, both bases are subject to the same base motion as shown in Fig. 42. At rotor speed of 35,000 rpm, the bearing coefficients are:

```
Brg Coefficients at 35,000 rpm

No. Kxx        Kxy        Kyx        Kyy        Cxx        Cxy        Cyx        Cyy
 1  335633.    183353.    -297550.   477252.    124.990    -3.22163   -3.22163   168.538
 2  197434.    .00000     0.00000    197434.    109.352    0.00000    0.00000    109.352
```

Again, for the comparison purposes, the equivalent steady state harmonic excitations at the frequency of 22,500 cpm are calculated as shown in Fig. 43. Note that, the base motion at station 2 only has the Y movement, but the steady state excitations exist in both X and Y directions due to the coupled bearing stiffness and damping coefficients at station 2. The absolute response for both base motion and steady state harmonic excitation are listed for comparison.

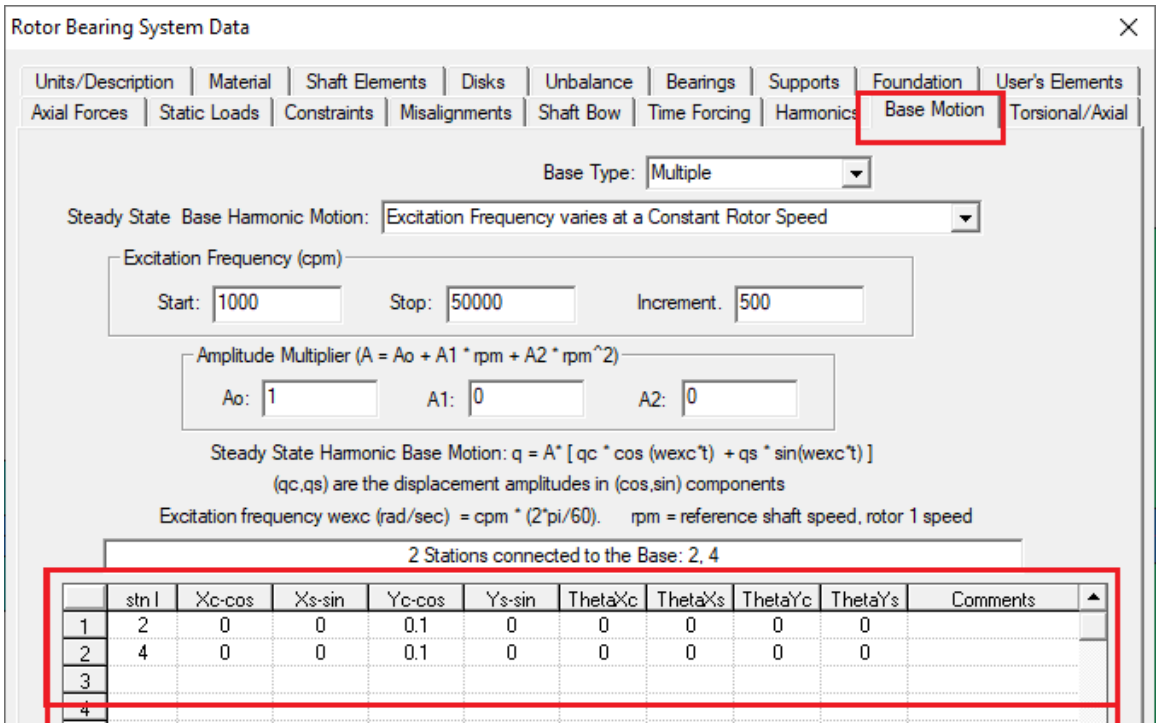


Fig. 42 – Base Motion

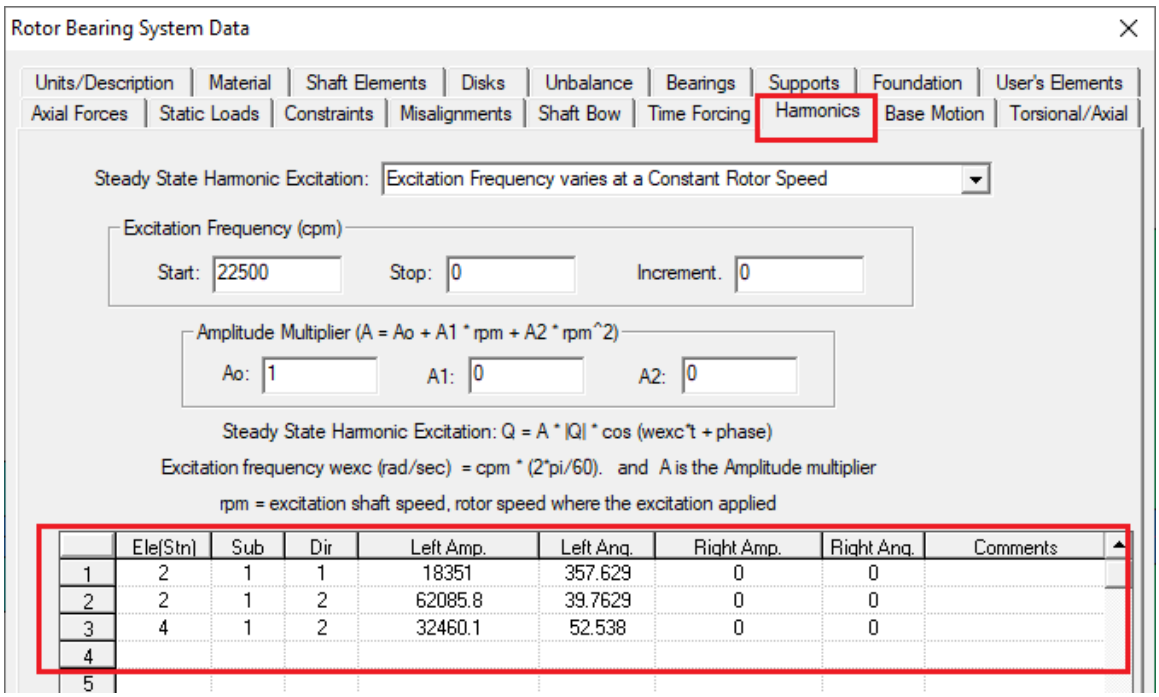


Fig. 43 – Equivalent Harmonic Excitation

************** Harmonic Response due to Base Motion (Excitation) **************
***** Excitation Frequency = 22500. cpm *****

Shaft 1 Speed= 35000.00 rpm = 3665.19 R/S = 583.33 Hz

*********************** Shaft Element Displacements ************************

		X		Y		Elliptical Orbit Data		
stn	sub	Amplitude	Phase	Amplitude	Phase	A	B	G
1	1	0.229E-01	163.5	0.147E+00	353.1	0.149E+00	-0.377E-02	99.
	2	0.216E-01	163.4	0.144E+00	353.5	0.145E+00	-0.373E-02	98.
	3	0.163E-01	163.0	0.131E+00	355.2	0.132E+00	-0.344E-02	97.
	4	0.111E-01	162.0	0.119E+00	357.4	0.120E+00	-0.293E-02	95.
2	1	0.858E-02	160.9	0.114E+00	358.6	0.114E+00	-0.259E-02	94.
	2	0.144E-02	131.8	0.994E-01	2.9	0.994E-01	-0.112E-02	91.
	3	0.785E-03	42.0	0.964E-01	4.4	0.964E-01	-0.478E-03	90.
3	1	0.575E-02	345.5	0.872E-01	10.2	0.873E-01	0.239E-02	87.
	2	0.110E-01	340.6	0.792E-01	17.7	0.797E-01	0.661E-02	84.
	3	0.116E-01	340.3	0.784E-01	18.7	0.789E-01	0.715E-02	83.
4	1	0.198E-01	325.9	0.770E-01	40.0	0.772E-01	0.190E-01	86.
5	1	0.302E-01	306.2	0.103E+00	62.4	0.104E+00	0.269E-01	98.
	2	0.451E-01	290.6	0.149E+00	75.9	0.154E+00	0.249E-01	104.
6	1	0.563E-01	283.6	0.184E+00	81.1	0.191E+00	0.207E-01	106.
	2	0.621E-01	281.0	0.202E+00	83.1	0.210E+00	0.184E-01	106.
7	1	0.650E-01	279.9	0.211E+00	83.9	0.220E+00	0.172E-01	107.

****

**************** Harmonic Response due to Shaft (1) Excitation ****************
***** Excitation Frequency = 22500. cpm *****

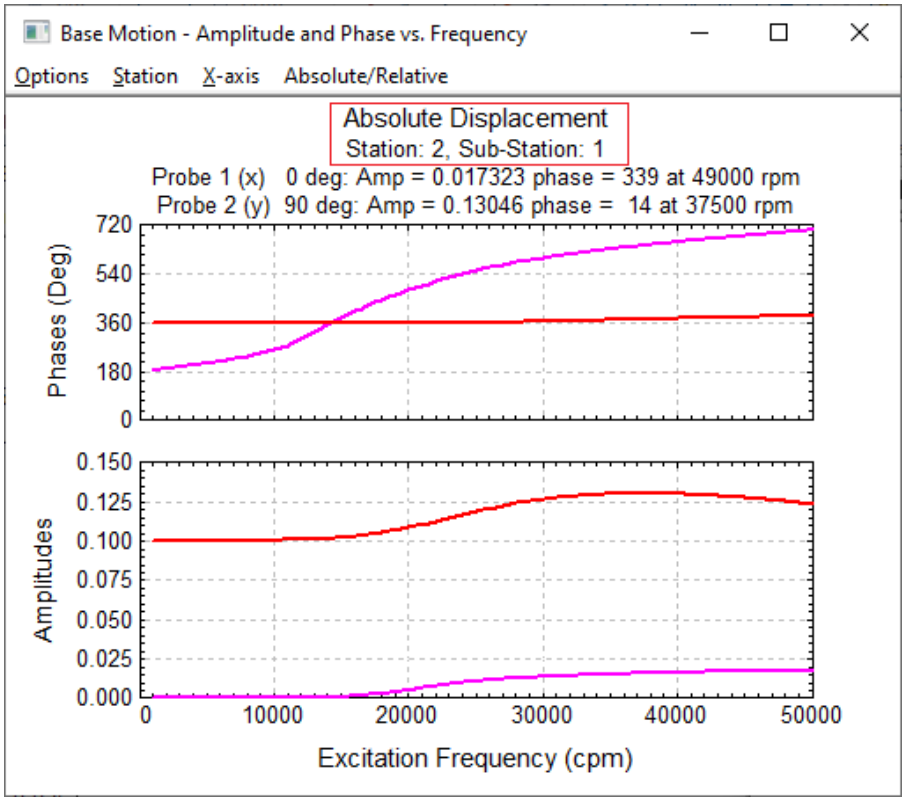
Shaft 1 Speed= 35000.00 rpm = 3665.19 R/S = 583.33 Hz

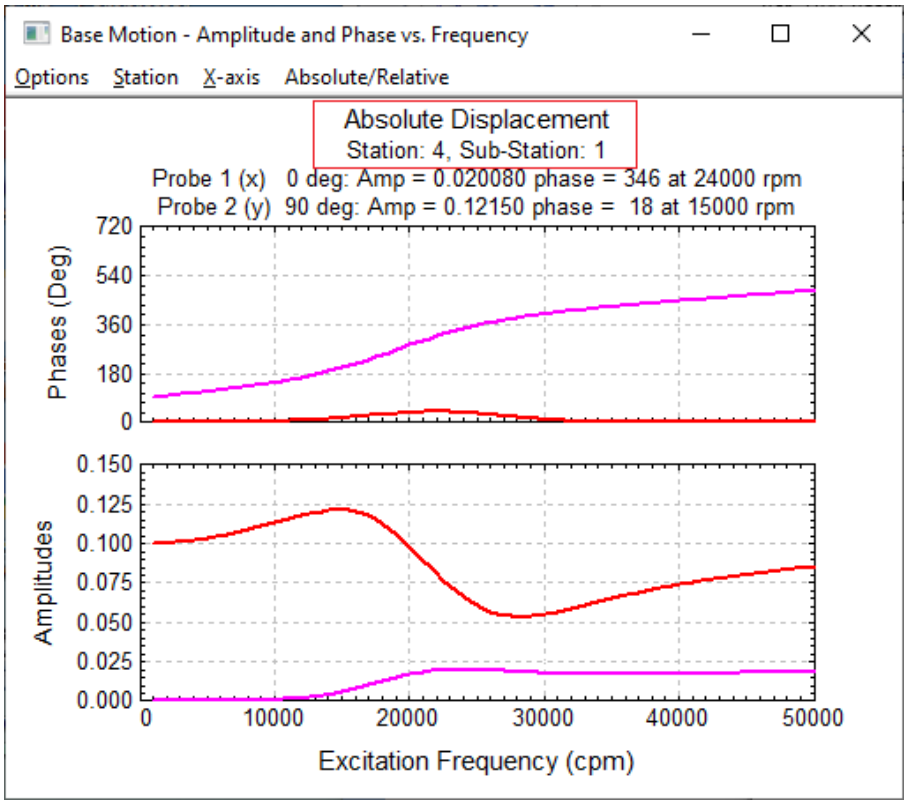
*********************** Shaft Element Displacements ************************

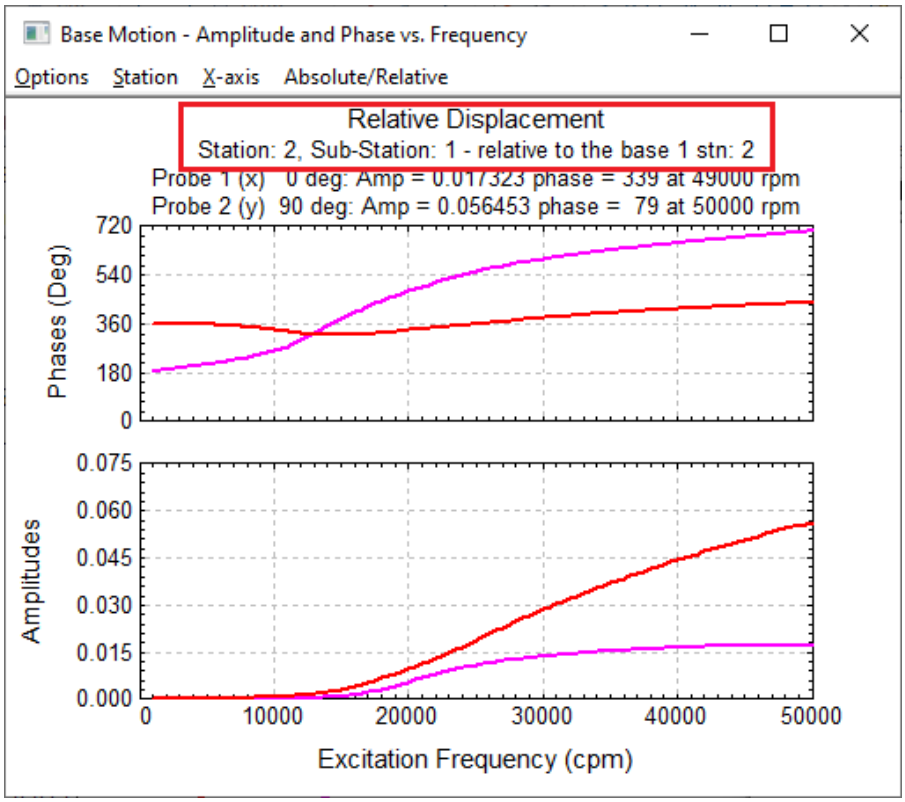
		X		Y		Elliptical Orbit Data		
stn	sub	Amplitude	Phase	Amplitude	Phase	A	B	G
1	1	0.229E-01	163.5	0.147E+00	353.1	0.149E+00	-0.377E-02	99.
	2	0.216E-01	163.4	0.144E+00	353.5	0.145E+00	-0.373E-02	98.
	3	0.163E-01	163.0	0.131E+00	355.2	0.132E+00	-0.344E-02	97.
	4	0.111E-01	162.0	0.119E+00	357.4	0.120E+00	-0.293E-02	95.
2	1	0.858E-02	160.9	0.114E+00	358.6	0.114E+00	-0.259E-02	94.
	2	0.144E-02	131.8	0.994E-01	2.9	0.994E-01	-0.112E-02	91.
	3	0.785E-03	42.0	0.964E-01	4.4	0.964E-01	-0.479E-03	90.
3	1	0.575E-02	345.5	0.872E-01	10.2	0.873E-01	0.239E-02	87.
	2	0.110E-01	340.6	0.792E-01	17.7	0.797E-01	0.661E-02	84.
	3	0.116E-01	340.3	0.784E-01	18.7	0.789E-01	0.715E-02	83.
4	1	0.198E-01	325.9	0.770E-01	40.0	0.772E-01	0.190E-01	86.
5	1	0.302E-01	306.2	0.103E+00	62.4	0.104E+00	0.269E-01	98.
	2	0.451E-01	290.6	0.149E+00	75.9	0.154E+00	0.249E-01	104.
6	1	0.563E-01	283.6	0.184E+00	81.1	0.191E+00	0.207E-01	106.
	2	0.621E-01	281.0	0.202E+00	83.1	0.210E+00	0.184E-01	106.
7	1	0.650E-01	279.9	0.211E+00	83.9	0.220E+00	0.172E-01	107.

****

The responses at both bearings for the base motion are shown below;







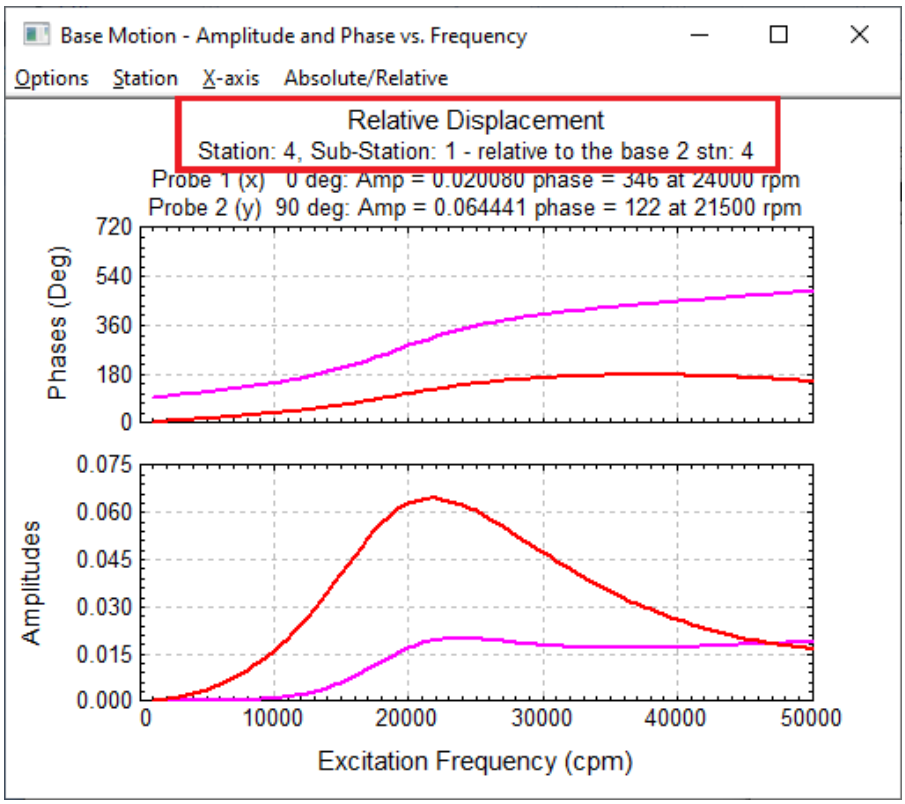
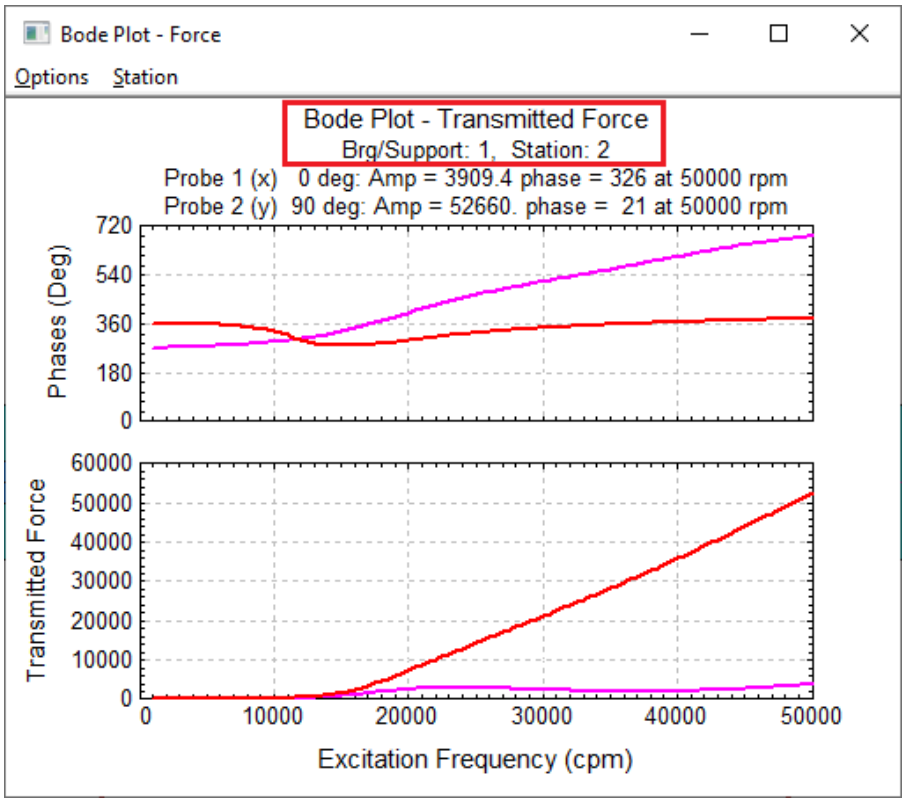


Fig. 44 – Absolute and Relative Displacements at Bearings



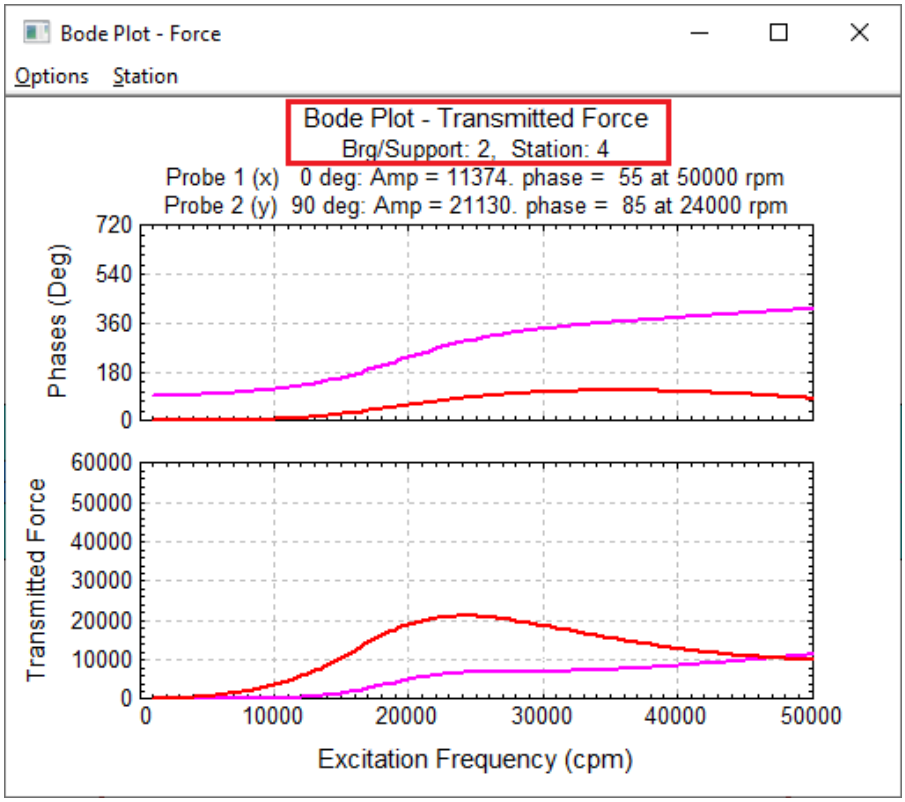


Fig. 45 – The Transmitted Bearing Forces due to Base Motion

Case 3: BaseMotion_4c.rot

In Case 2, although Multiple Bases are used, two bases have the same base motion. So, in this case, a Single base model is used as shown in Fig. 46. The results for the base motion in this case are identical to the results in Case 2 and are not repeated here.

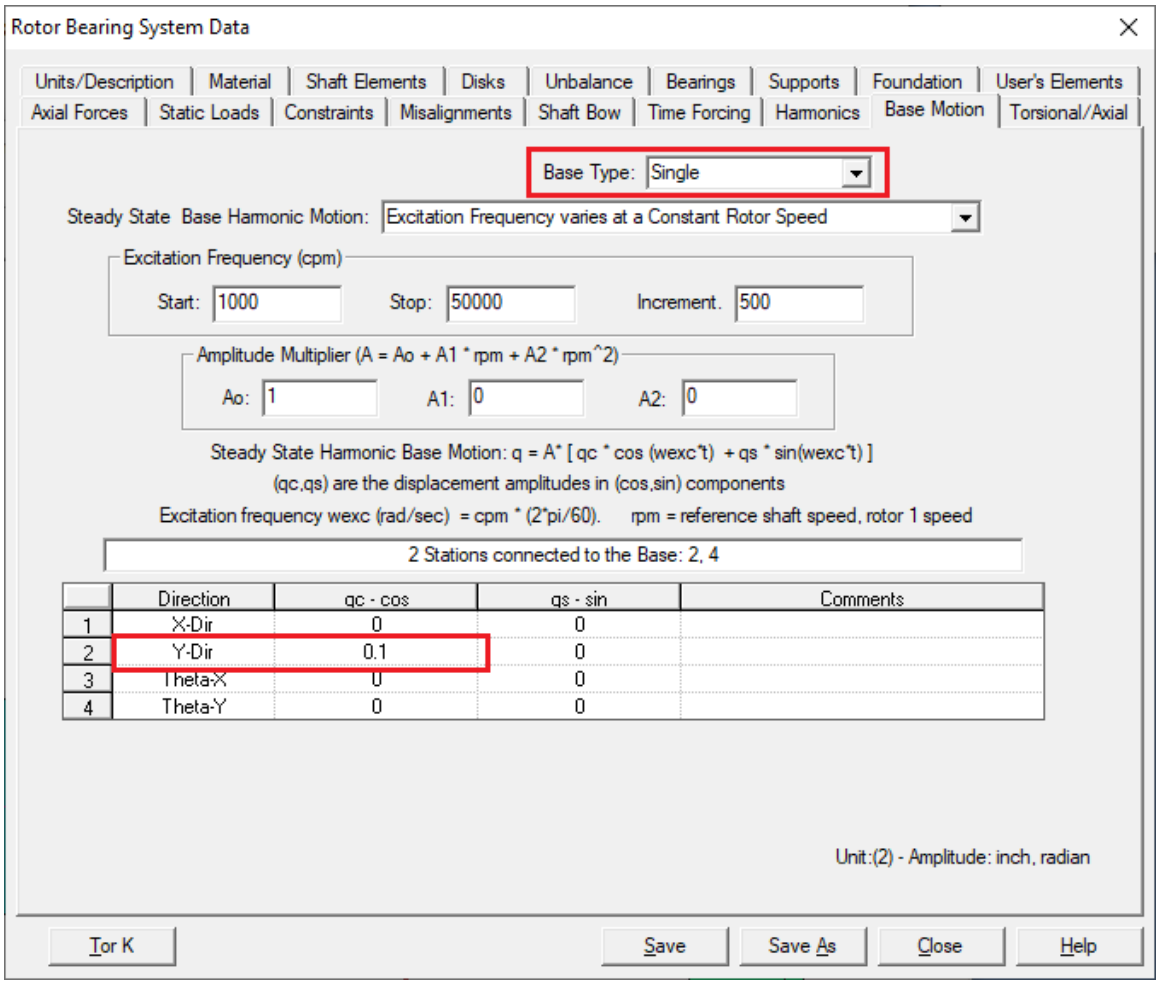


Fig. 46 – Base Motion – Single Base

Case 4: BaseMotion_4d.rot

In this case, bearing #2 (station 4) is connected to a support (station 8) as shown in Fig. 47. The base motion is acting on the stations 2 and 8.

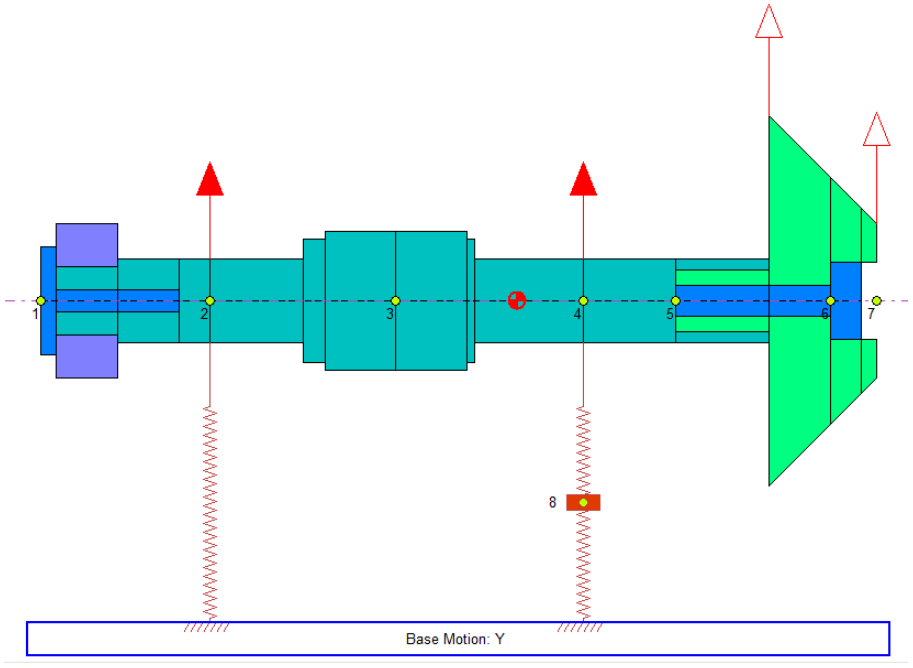


Fig. 47 – System Model

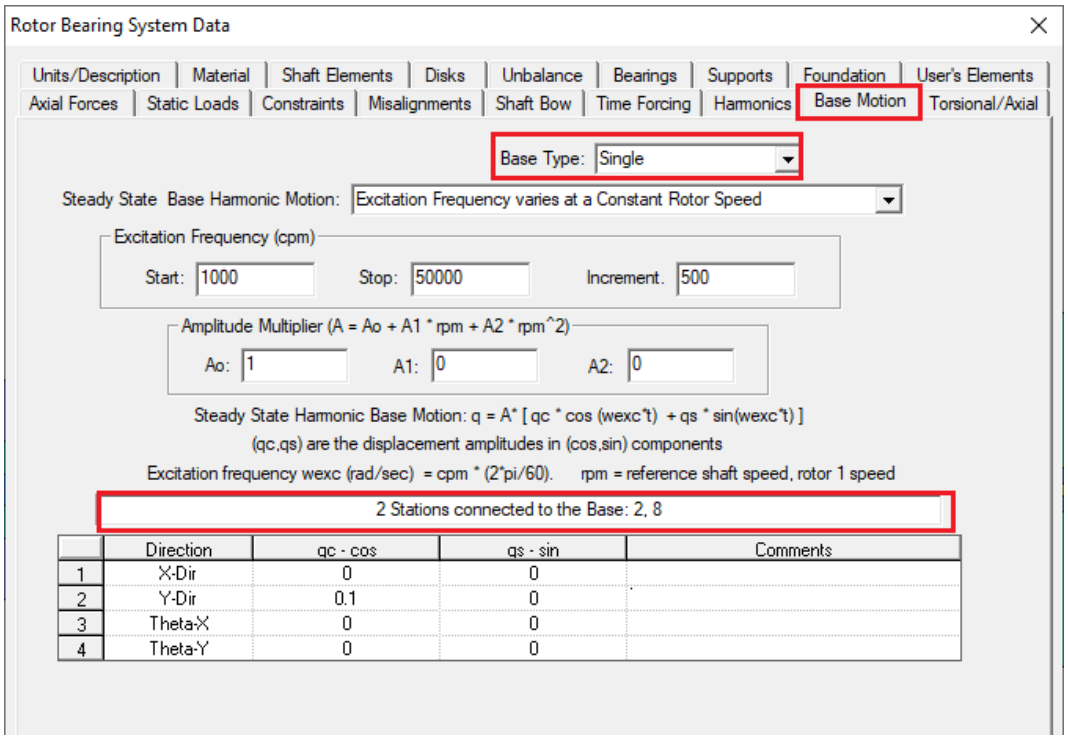
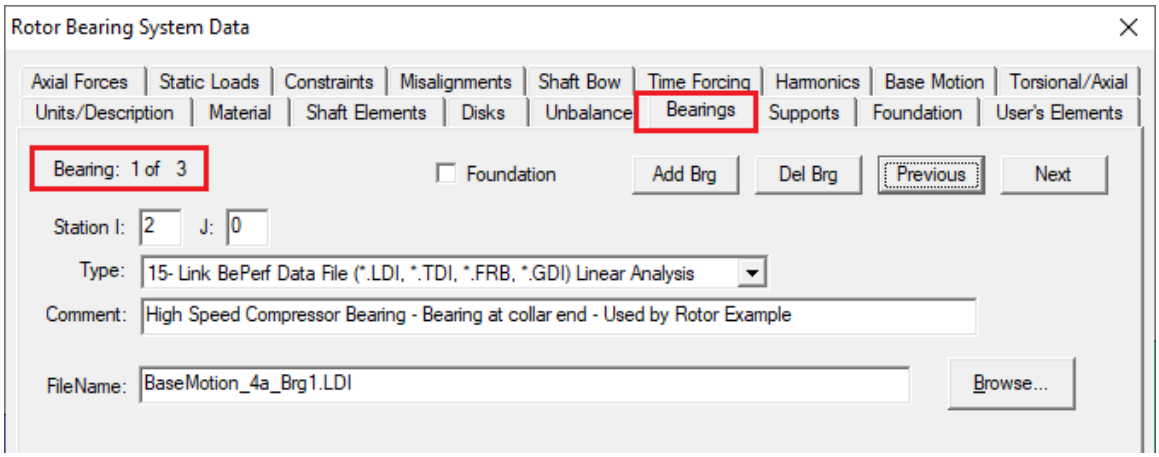
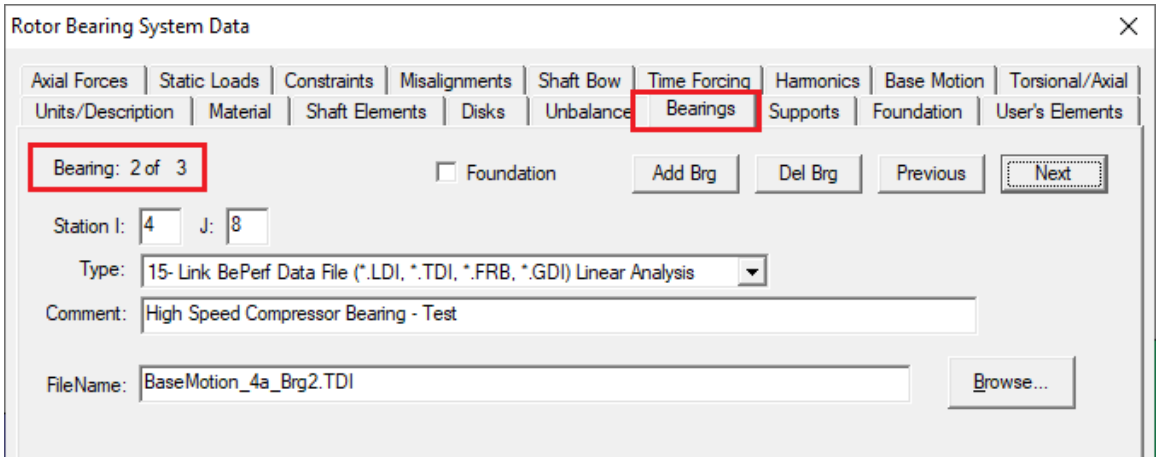
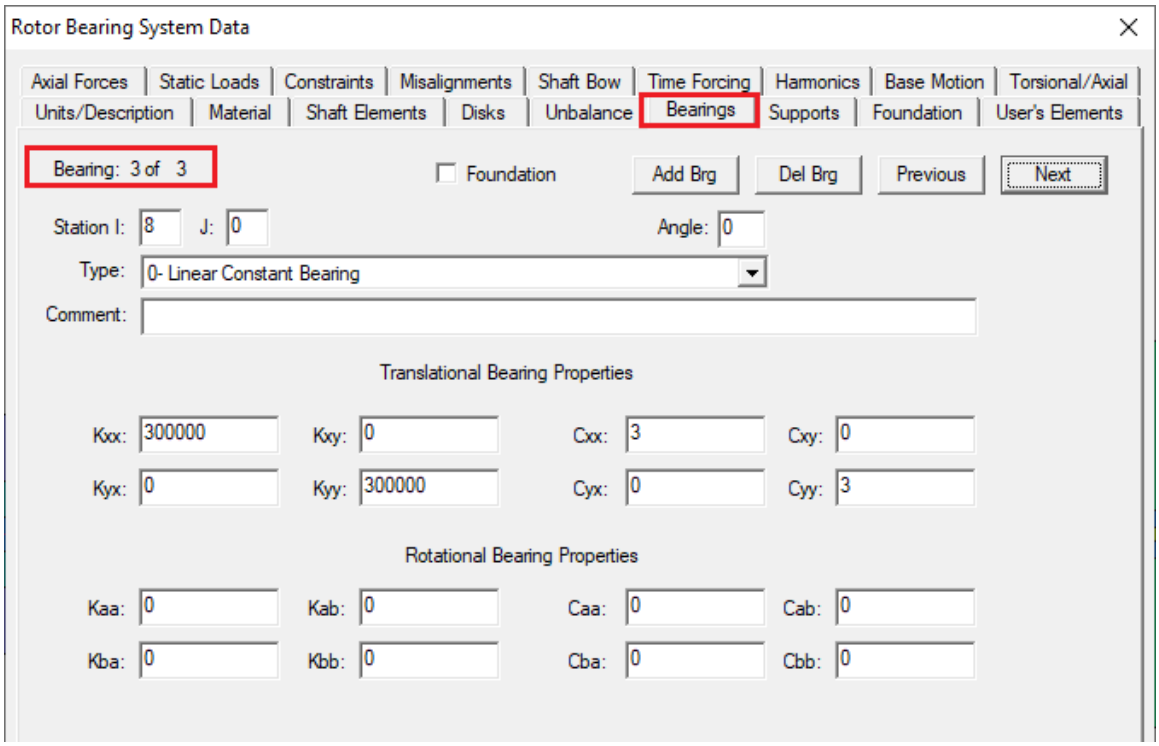


Fig. 48 – Base Motion


Rotor Bearing System Data
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Bearing: 1 of 3
Foundation
Add Brg
Del Brg
Previous
Next
Station I: 2
J: 0
Type: 15- Link BePerf Data File (*.LDI, *.TDI, *.FRB, *.GDI) Linear Analysis
Comment: High Speed Compressor Bearing - Bearing at collar end - Used by Rotor Example
FileName: BaseMotion_4a_Brg1.LDI
Browse...


Rotor Bearing System Data
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Bearing: 2 of 3
Foundation
Add Brg
Del Brg
Previous
Next
Station I: 4
J: 8
Type: 15- Link BePerf Data File (*.LDI, *.TDI, *.FRB, *.GDI) Linear Analysis
Comment: High Speed Compressor Bearing - Test
FileName: BaseMotion_4a_Brg2.TDI
Browse...


Rotor Bearing System Data
Axial Forces
Static Loads
Constraints
Misalignments
Shaft Bow
Time Forcing
Harmonics
Base Motion
Torsional/Axial
Units/Description
Material
Shaft Elements
Disks
Unbalance
Bearings
Supports
Foundation
User's Elements
Bearing: 3 of 3
Foundation
Add Brg
Del Brg
Previous
Next
Station I: 8
J: 0
Angle: 0
Type: 0- Linear Constant Bearing
Comment:
Translational Bearing Properties
Kxx: 300000
Kxy: 0
Cxx: 3
Cxy: 0
Kyx: 0
Kyy: 300000
Cyx: 0
Cyy: 3
Rotational Bearing Properties
Kaa: 0
Kab: 0
Caa: 0
Cab: 0
Kba: 0
Kbb: 0
Cba: 0
Cbb: 0

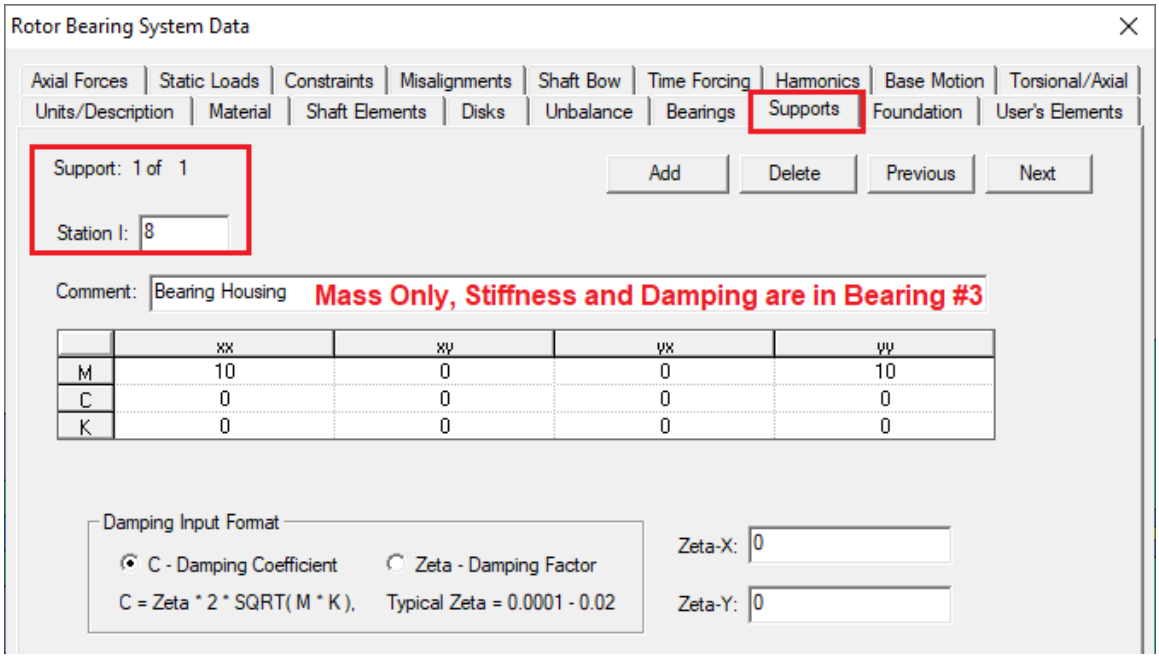
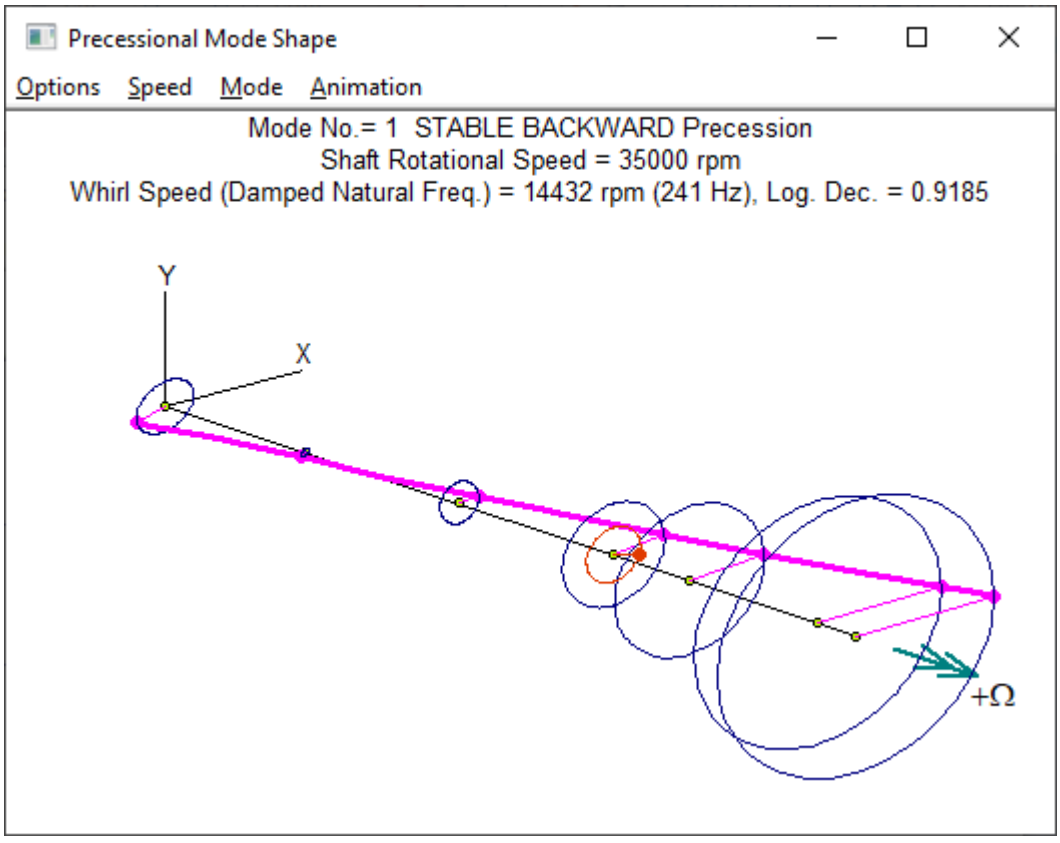
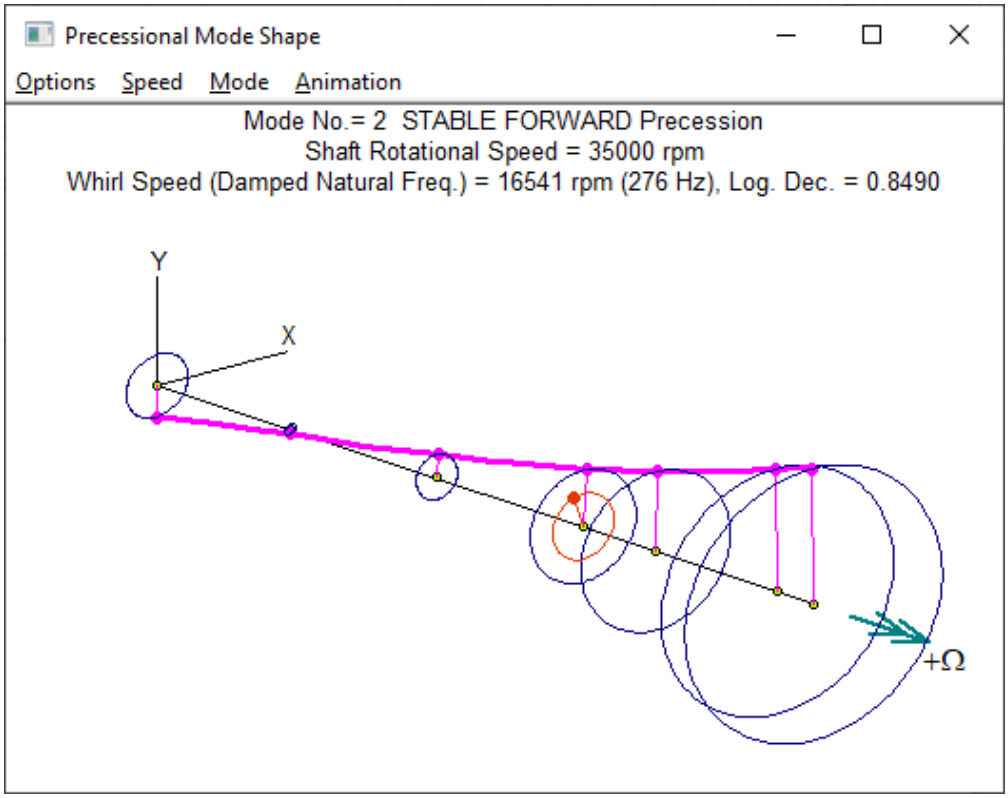
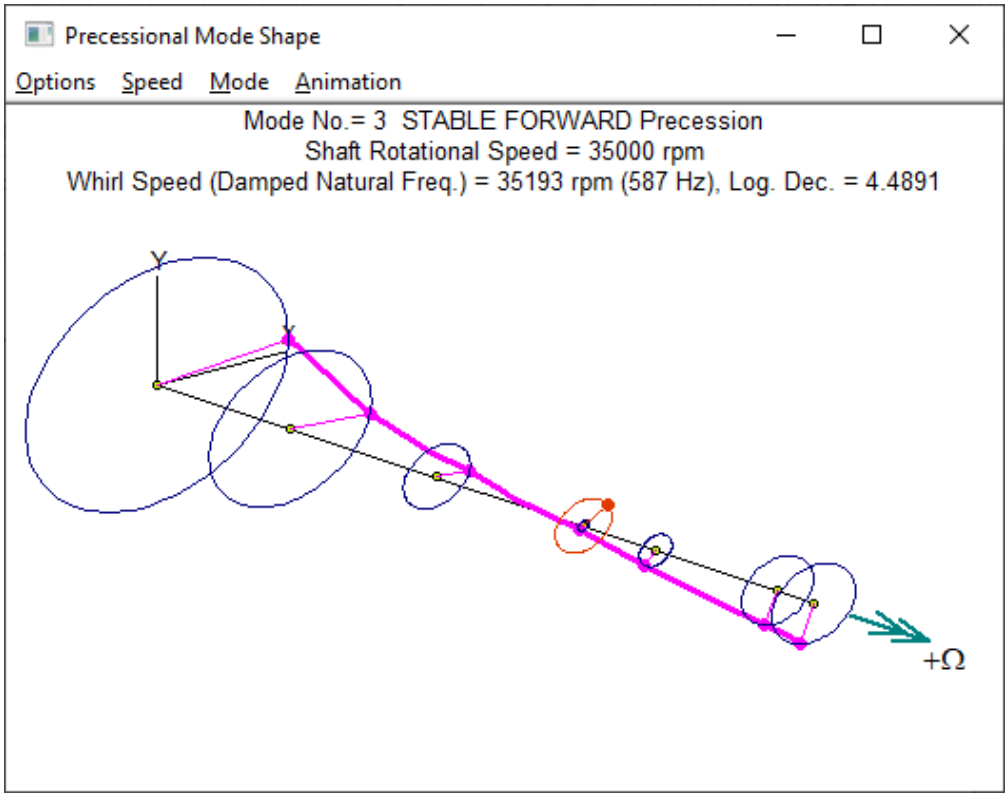


Fig. 49 – Bearing and support data

At the rotor speed of 35,000 rpm, the first five natural frequencies and modes are shown in Fig. 50.




Precessional Mode Shape
Options Speed Mode Animation
Mode No.= 2 STABLE FORWARD Precession
Shaft Rotational Speed = 35000 rpm
Whirl Speed (Damped Natural Freq.) = 16541 rpm (276 Hz), Log. Dec. = 0.8490
Y
X
+Ω


Precessional Mode Shape
Options Speed Mode Animation
Mode No.= 3 STABLE FORWARD Precession
Shaft Rotational Speed = 35000 rpm
Whirl Speed (Damped Natural Freq.) = 35193 rpm (587 Hz), Log. Dec. = 4.4891
Y
X
+Ω

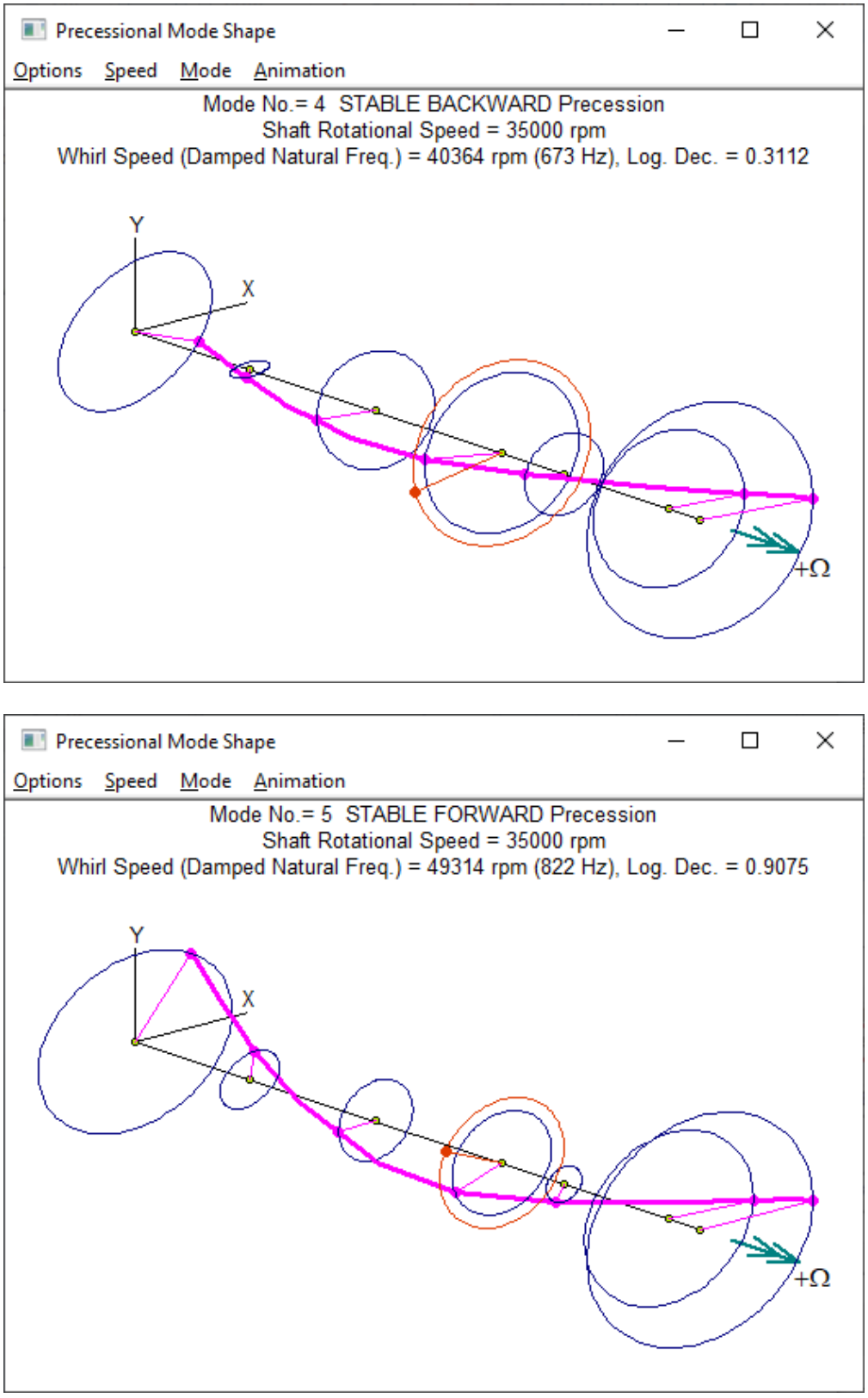
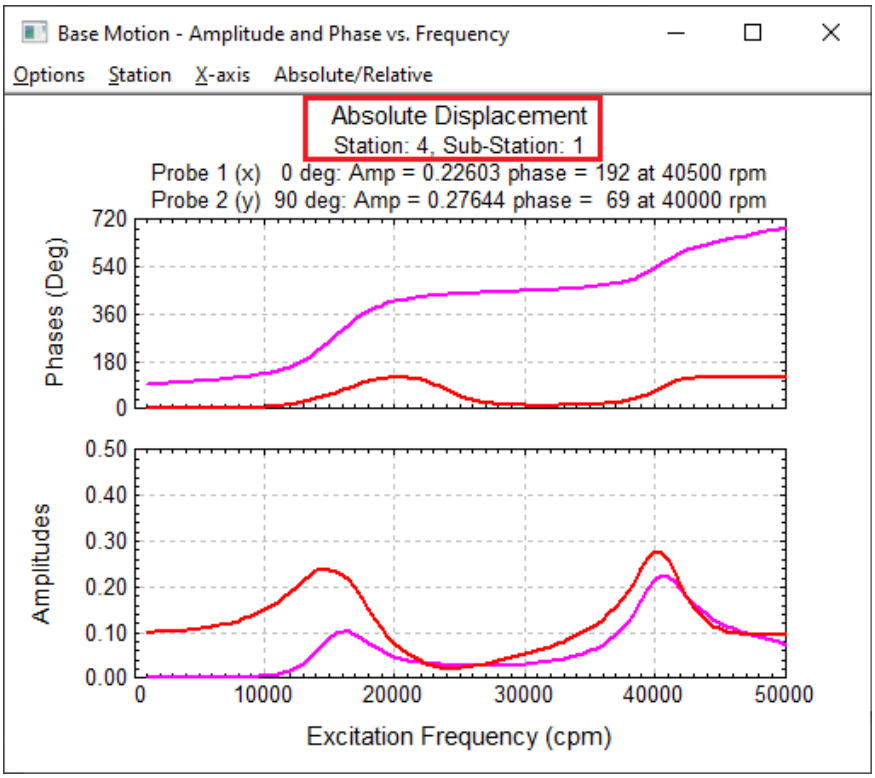
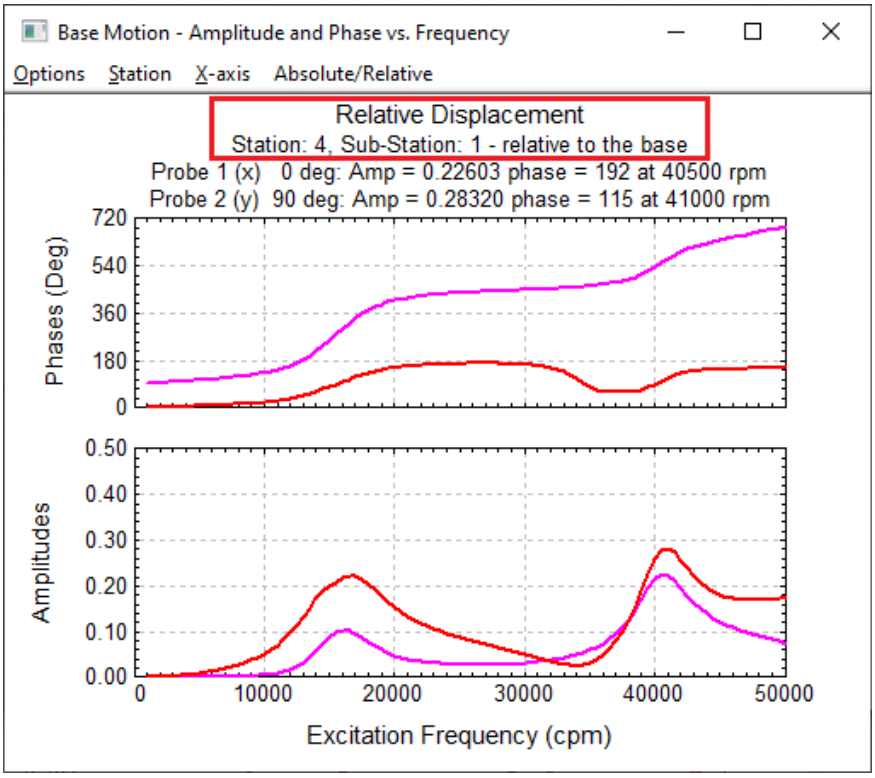


Fig. 50 – The first five natural frequencies and modes

The absolute and relative displacements at stations 4 and 8, bearing #2 and its support, are shown in Fig. 51.





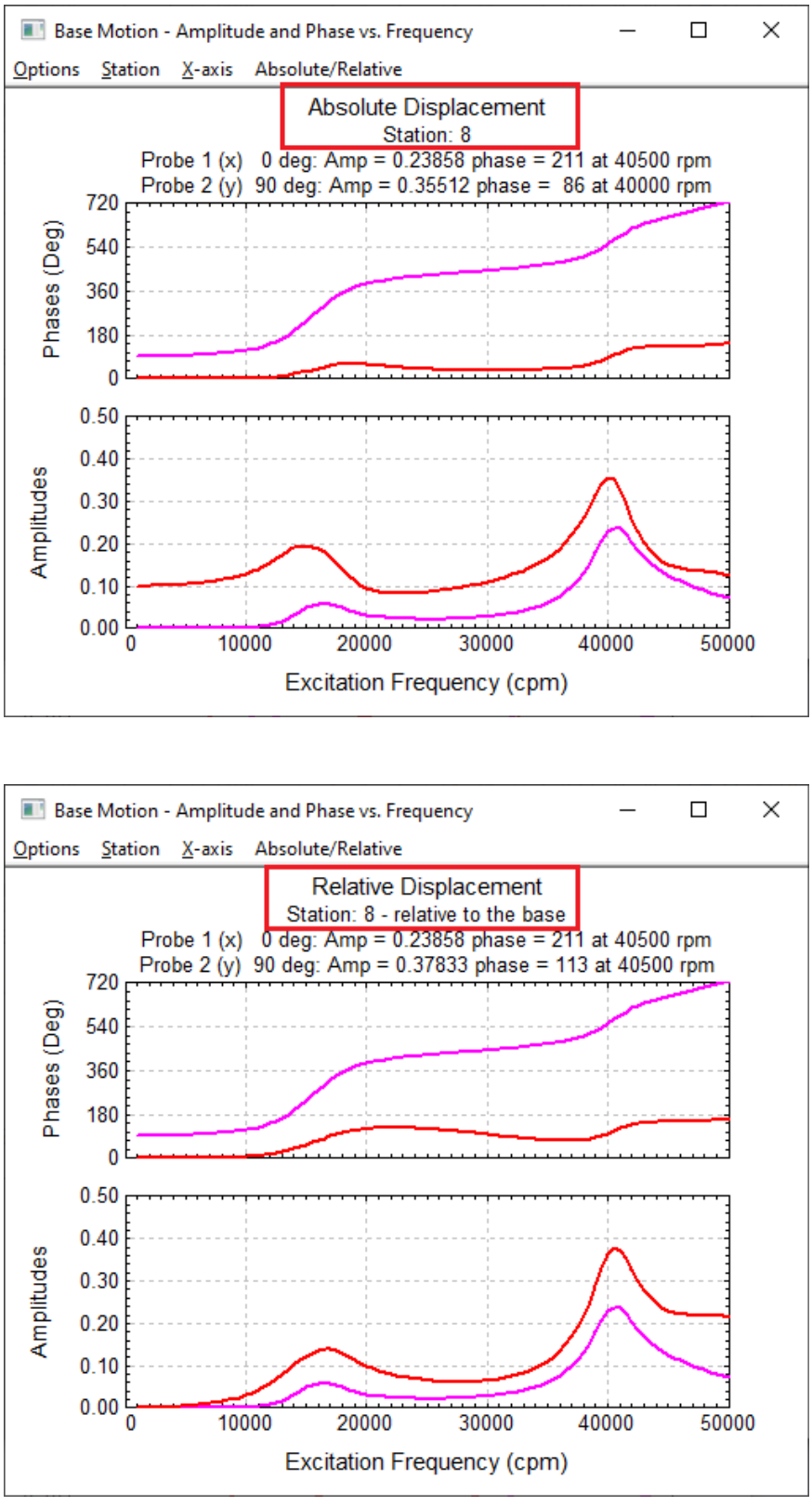
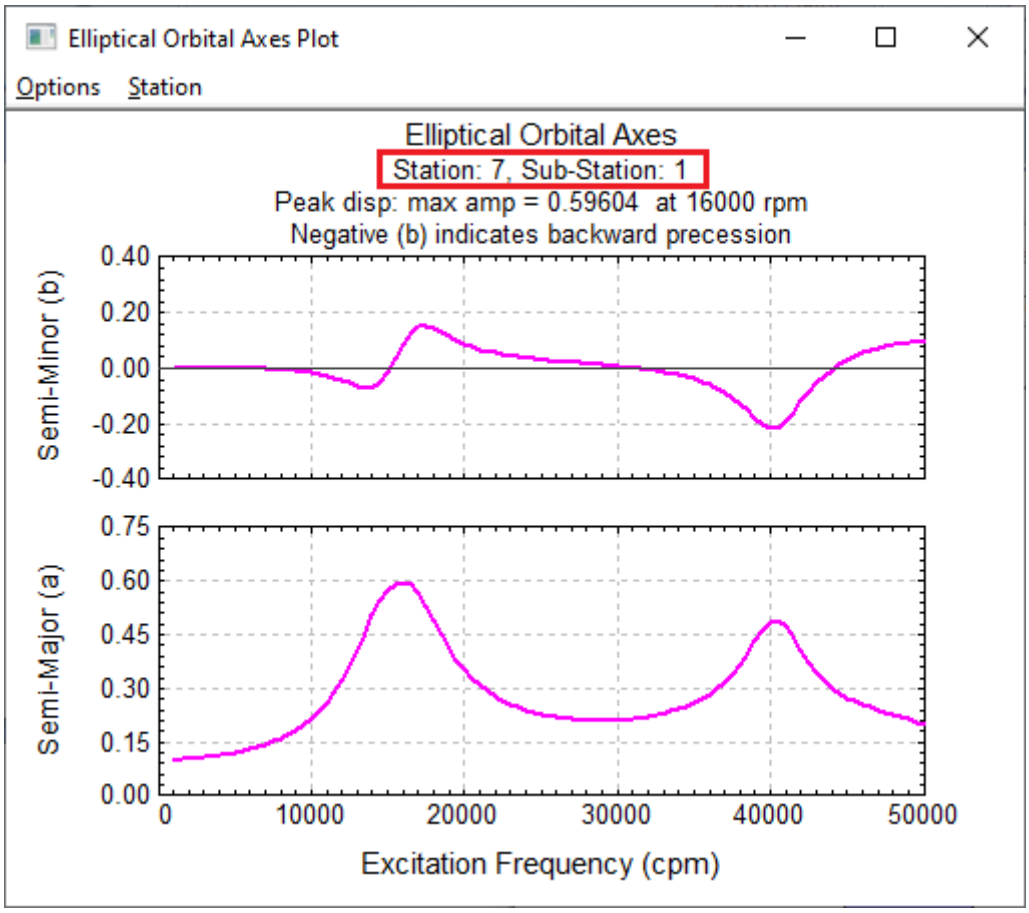


Fig. 51 – The Absolute and Relative Displacements at Stations 4 and 8

Again, let us examine the impeller station (station 7) where the maximum displacement occurs. It starts from with the base motion (a straight line motion), then backward precession when approaching the 1st mode (backward mode), after 15,000 cpm, the motion becomes forward and reaches the maximum peak at 16,000 cpm. From the previous Whirl Speed/Stability Analysis (Fig. 50), it shows that the 2nd mode (forward mode) has a slightly smaller damping (log. Decrement) than that of the 1st mode (backward mode). The second peak occurs around 40,000 cpm and the motion is a backward precession. This can also be observed from the mode frequency and damping at mode #4.



Case 5: BaseMotion_4e.rot

This case is identical to the previous Case 4, except the base motion has entered as multiple bases. The results are identical to Case 4, and not repeated here.

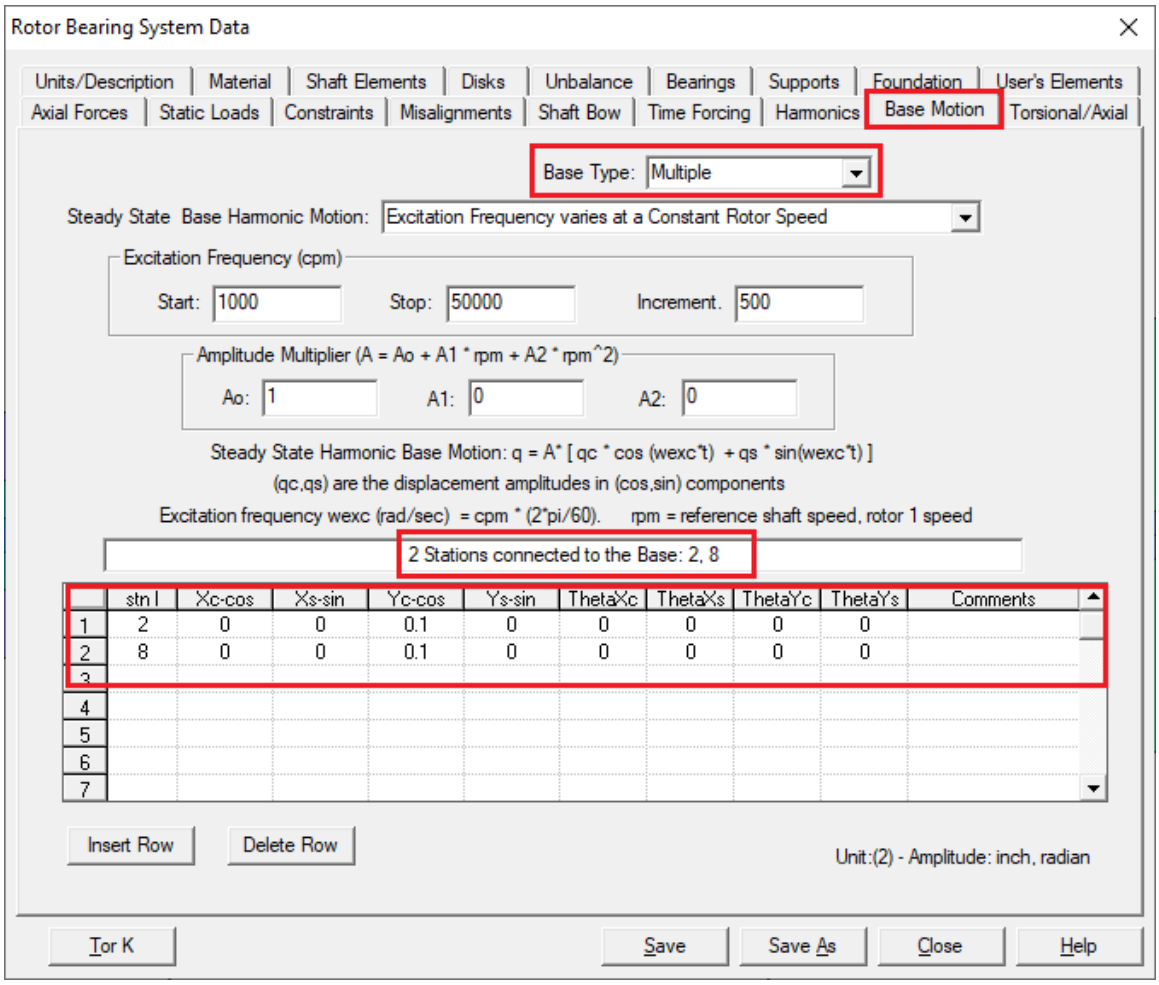


Fig. 52 – Base Motion – Multiple Bases

Let us consider a steady state harmonic excitation with a frequency of 40,000 cpm acting on the station 2 and station 8 at the rotor speed of 35,000 rpm. At 35.000 rpm, the bearing coefficients are:

```
Brg Coefficients at 35.000 rpm
Stn   Kxx         Kxy         Kyx         Kyy         Cxx         Cxy         Cyx         Cyy
 2   335633.     183353.     -297550.    477252.     124.990    -3.22163    -3.22163     168.538
 4   197434.     0.00000     0.00000     197434.     109.352     0.00000     0.00000     109.352
 8   300000.     0.00000     0.00000     300000.     3.00000     0.00000     0.00000     3.00000
```

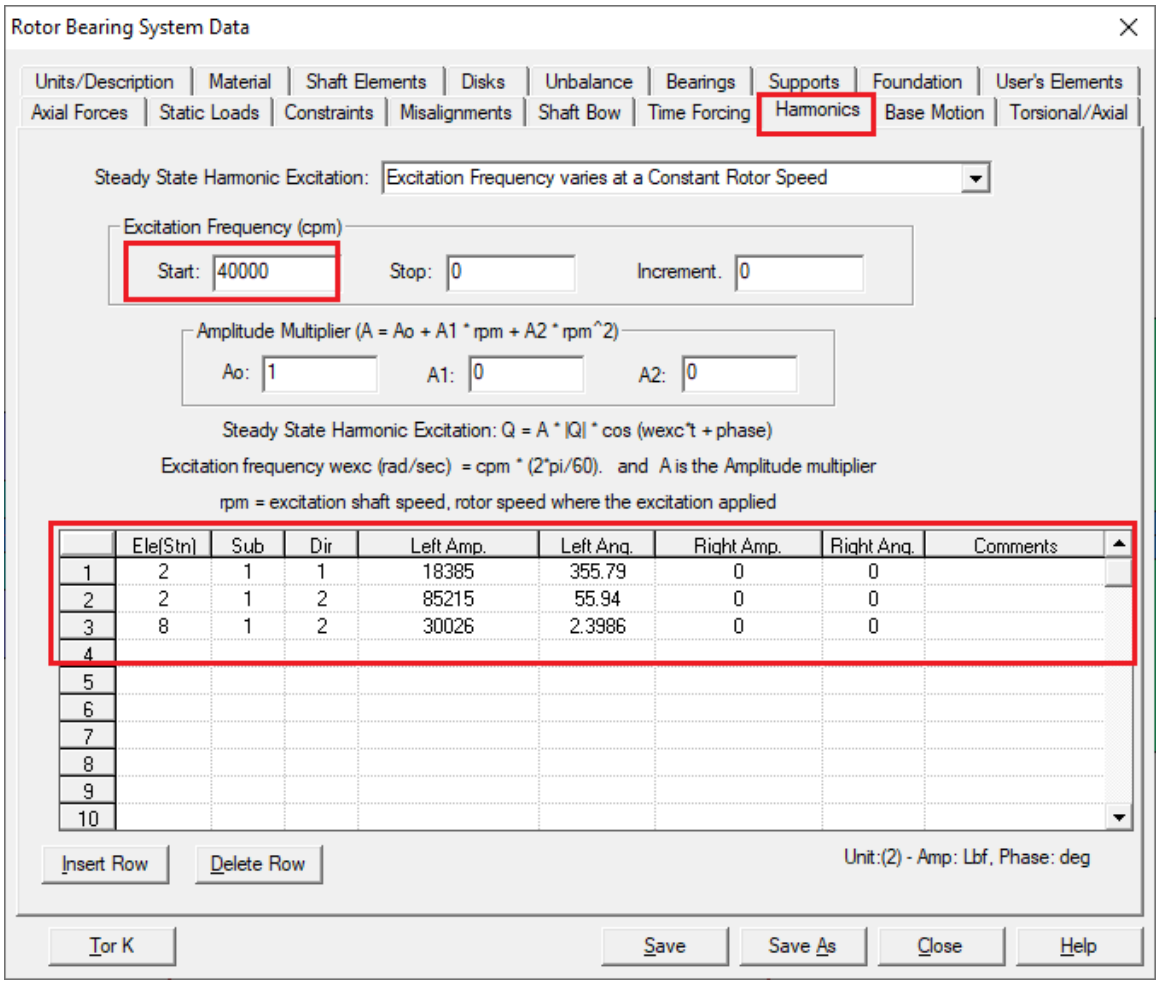


Fig. 53 – Steady State Harmonic Excitation at 40,000 cpm

Again, the absolute displacements can be compared with the results from the base motion.

```
************ Harmonic Response due to Base Motion (Excitation) *************
             *** Excitation Frequency =  40000.      cpm ***

  Shaft  1     Speed=   35000.00 rpm  =   3665.19 R/S  =     583.33 Hz

********************** Shaft Element Displacements ************************
         =====  X  =====     =====  Y  =====        Elliptical Orbit Data
 stn sub Amplitude Phase     Amplitude Phase        A          B          G

  1   1  0.241E+00  333.3    0.139E+00  298.6     0.269E+00  -0.709E-01     27.
      2  0.222E+00  332.3    0.125E+00  303.2     0.249E+00  -0.542E-01     28.
      3  0.148E+00  325.3    0.847E-01  334.8     0.170E+00   0.121E-01     30.
      4  0.810E-01  306.3    0.907E-01   20.4     0.978E-01   0.722E-01     56.
  2   1  0.582E-01  282.8    0.111E+00   35.7     0.114E+00   0.522E-01    105.
      2  0.893E-01  204.0    0.190E+00   55.7     0.205E+00  -0.434E-01    113.
      3  0.104E+00  198.0    0.205E+00   57.7     0.222E+00  -0.616E-01    113.
  3   1  0.154E+00  186.9    0.253E+00   62.5     0.271E+00  -0.118E+00    114.
      2  0.201E+00  181.5    0.293E+00   65.6     0.313E+00  -0.170E+00    114.
      3  0.206E+00  181.1    0.297E+00   65.9     0.316E+00  -0.175E+00    114.
  4   1  0.216E+00  175.9    0.276E+00   69.2     0.290E+00  -0.197E+00    115.
```

5	1	0.125E+00	165.7	0.124E+00	76.4	0.125E+00	-0.124E+00	35.
	2	0.651E-01	38.1	0.129E+00	236.5	0.143E+00	-0.185E-01	116.
6	1	0.189E+00	13.0	0.312E+00	242.9	0.340E+00	-0.133E+00	115.
	2	0.254E+00	9.9	0.404E+00	243.9	0.438E+00	-0.190E+00	115.
7	1	0.287E+00	8.8	0.450E+00	244.3	0.487E+00	-0.219E+00	115.

*** Flexible Support Displacements

8		0.228E+00	194.0	0.355E+00	86.4	0.365E+00	-0.211E+00	107.

****

********** Harmonic Response due to Shaft (1) Excitation **********
*** Excitation Frequency = 40000. cpm ***

Shaft 1 Speed= 35000.00 rpm = 3665.19 R/S = 583.33 Hz

********** Shaft Element Displacements **********

		X		Y		Elliptical Orbit Data		
stn	sub	Amplitude	Phase	Amplitude	Phase	A	B	G
1	1	0.241E+00	333.3	0.139E+00	298.6	0.269E+00	-0.708E-01	27.
	2	0.222E+00	332.3	0.125E+00	303.2	0.249E+00	-0.542E-01	28.
	3	0.148E+00	325.3	0.847E-01	334.8	0.170E+00	0.121E-01	30.
	4	0.810E-01	306.3	0.907E-01	20.4	0.978E-01	0.722E-01	56.
2	1	0.582E-01	282.8	0.111E+00	35.7	0.114E+00	0.522E-01	105.
	2	0.893E-01	204.0	0.190E+00	55.7	0.205E+00	-0.434E-01	113.
	3	0.104E+00	198.0	0.205E+00	57.7	0.222E+00	-0.616E-01	113.
3	1	0.154E+00	186.9	0.253E+00	62.5	0.271E+00	-0.118E+00	114.
	2	0.201E+00	181.5	0.293E+00	65.6	0.313E+00	-0.170E+00	114.
	3	0.206E+00	181.1	0.297E+00	65.9	0.316E+00	-0.175E+00	114.
4	1	0.216E+00	175.9	0.276E+00	69.2	0.290E+00	-0.197E+00	115.
5	1	0.125E+00	165.7	0.124E+00	76.4	0.125E+00	-0.124E+00	35.
	2	0.651E-01	38.1	0.129E+00	236.5	0.143E+00	-0.185E-01	116.
6	1	0.189E+00	13.0	0.312E+00	242.9	0.340E+00	-0.133E+00	115.
	2	0.254E+00	9.9	0.404E+00	243.9	0.438E+00	-0.190E+00	115.
7	1	0.287E+00	8.8	0.450E+00	244.3	0.487E+00	-0.219E+00	115.

*** Flexible Support Displacements

8		0.228E+00	194.0	0.355E+00	86.4	0.365E+00	-0.211E+00	107.

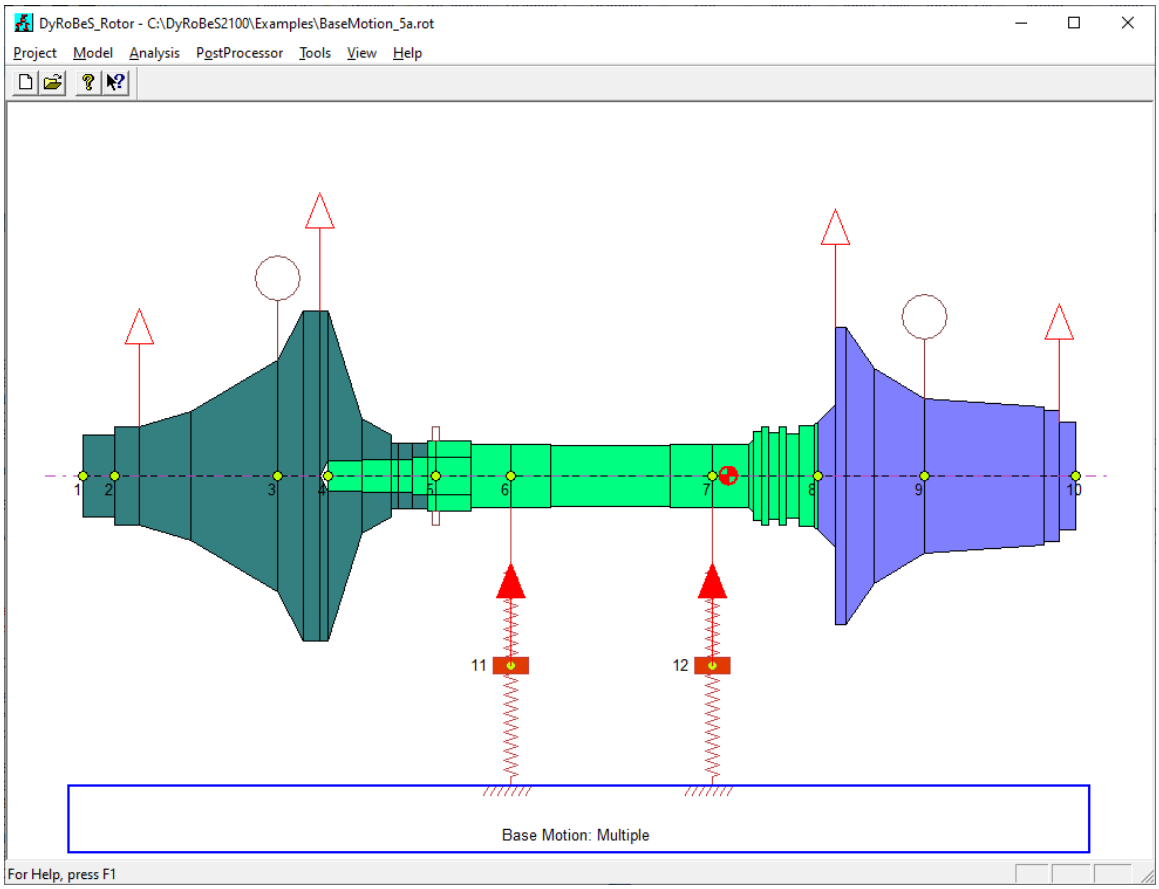
****

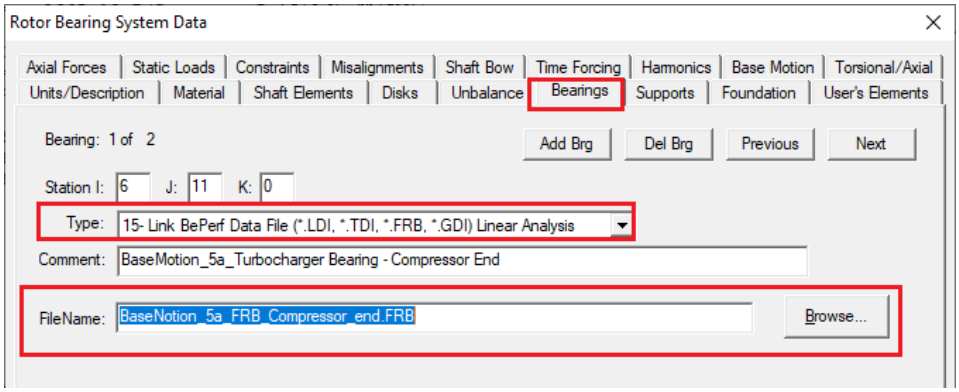
Example 5 – Turbocharger

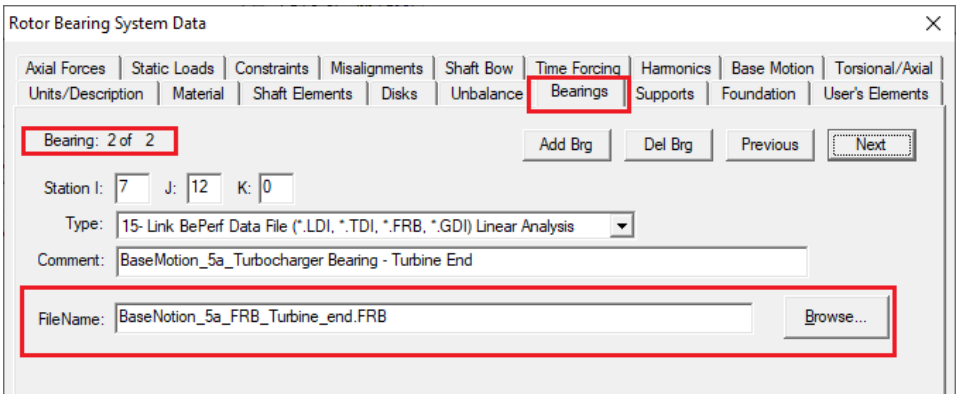
An automobile turbocharger is used in this demonstration. All the automobile turbochargers with floating ring bearings are operated beyond the instability threshold in linear theory and nonlinear analysis is required. Currently, in Ver 21, the Base Motion Analysis is a linear analysis for a linear system. The floating ring bearing data is save in BePerf and read directly in Rotor program. Linearized bearing coefficients are calculated first before the base motion is performed. Since this is a linear analysis, it may not give you the accurate prediction of the rotor behavior, however, it certainly give you some idea on how the rotor will behave for the base motion with the specified excitations.

Case 1: BaseMotion_5a.rot

The rotor model and some inputs are shown below:







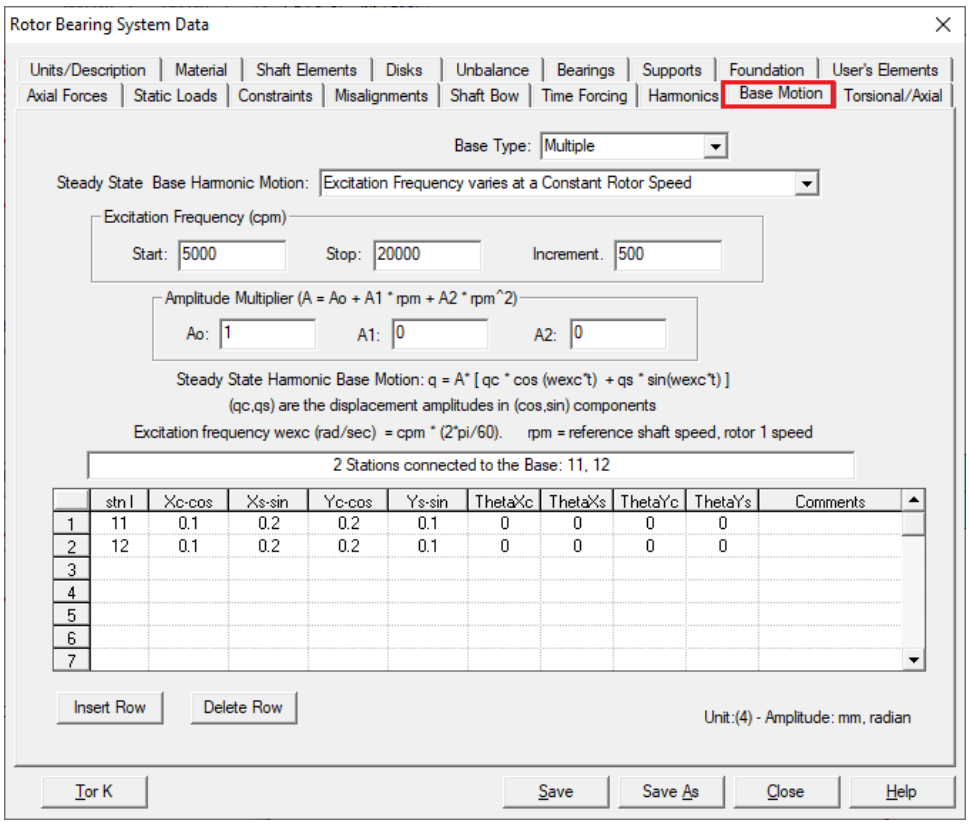


Fig. 54 – Turbocharger Model

The base motion analysis is performed at a rotor speed of 125,000 rpm with excitation frequency from 5000 to 20,000 cpm.

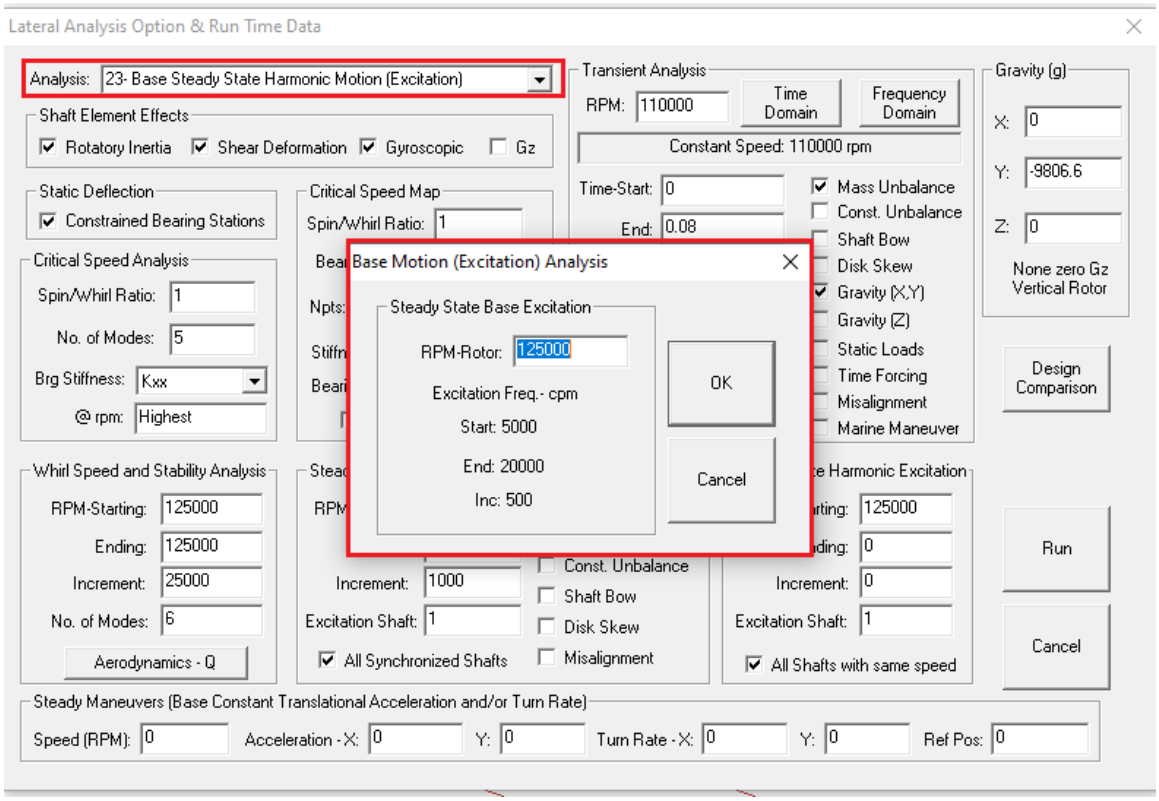
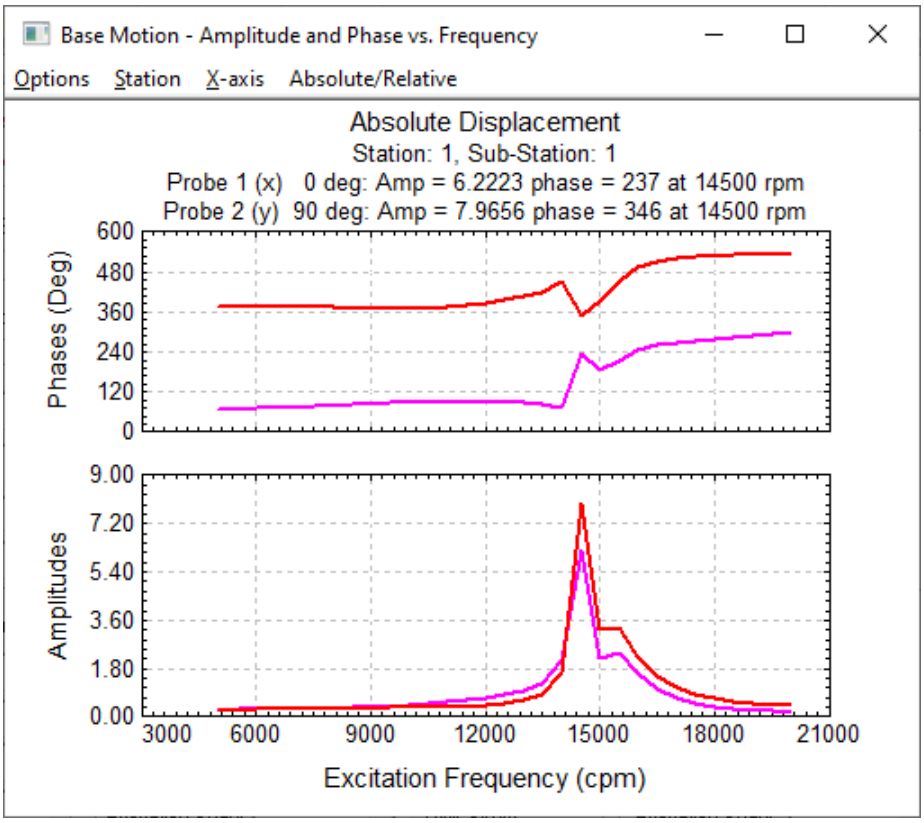
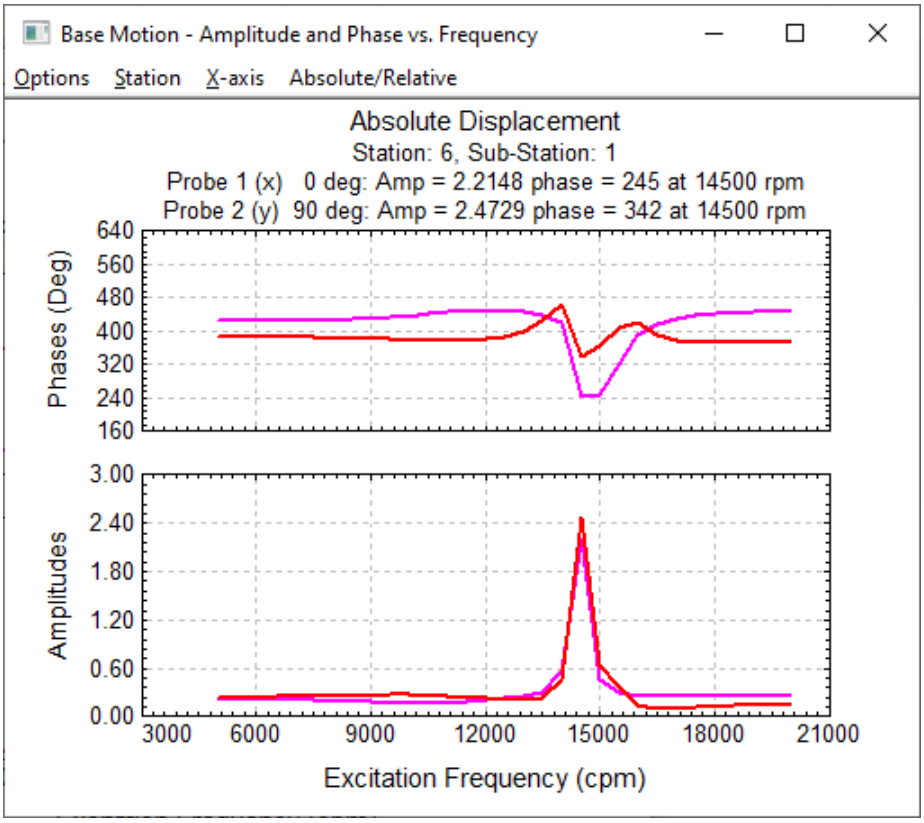
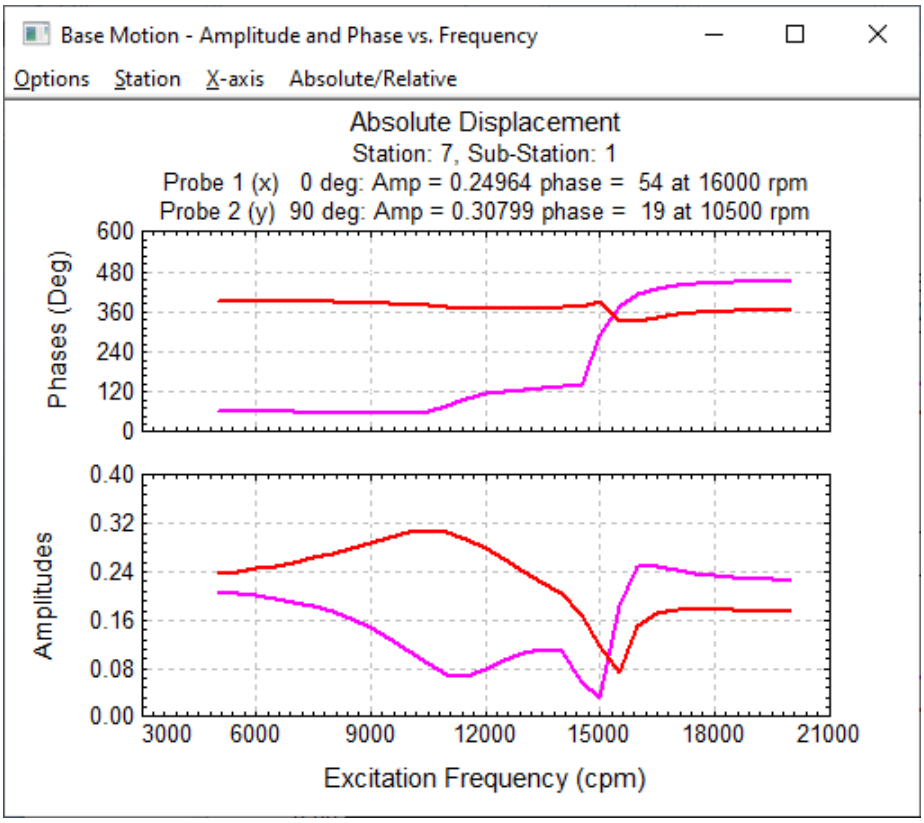


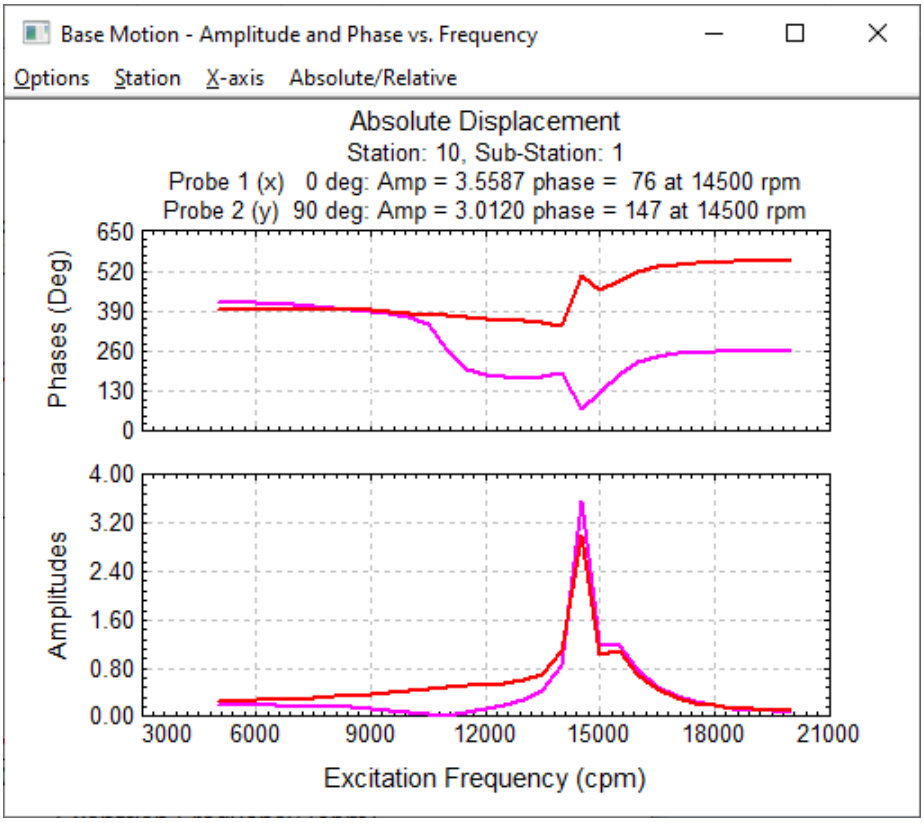
Fig. 54 – Base Motion Analysis

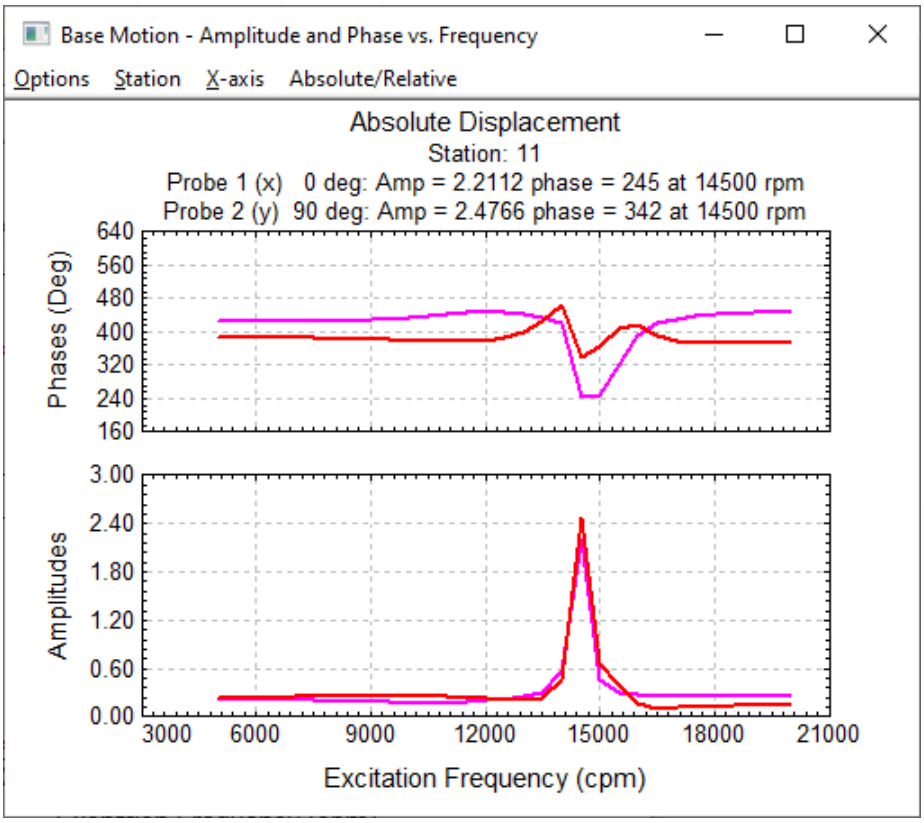
The responses due to the base motion at some stations are shown below. To further understand the rotor behavior, a whirl speed analysis is performed at the rotor 125,000 rpm, it clearly shows that a precessional mode with a frequency of 14,401 cpm is excited by this base motion and the floating ring at the turbine end is nearly stationary and the ring at the compressor end is very active. The mode shape is also shown for reference.

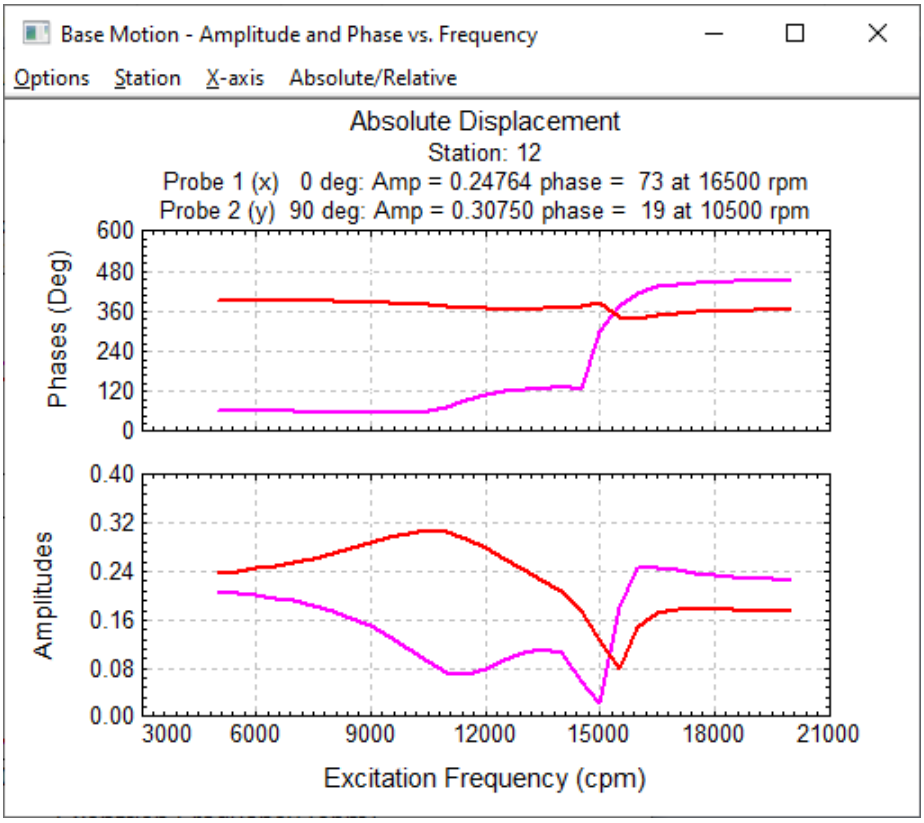

Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 1, Sub-Station: 1
Probe 1 (x) 0 deg: Amp = 6.2223 phase = 237 at 14500 rpm
Probe 2 (y) 90 deg: Amp = 7.9656 phase = 346 at 14500 rpm
Phases (Deg)
600
480
360
240
120
0
Amplitudes
9.00
7.20
5.40
3.60
1.80
0.00
3000
6000
9000
12000
15000
18000
21000
Excitation Frequency (cpm)

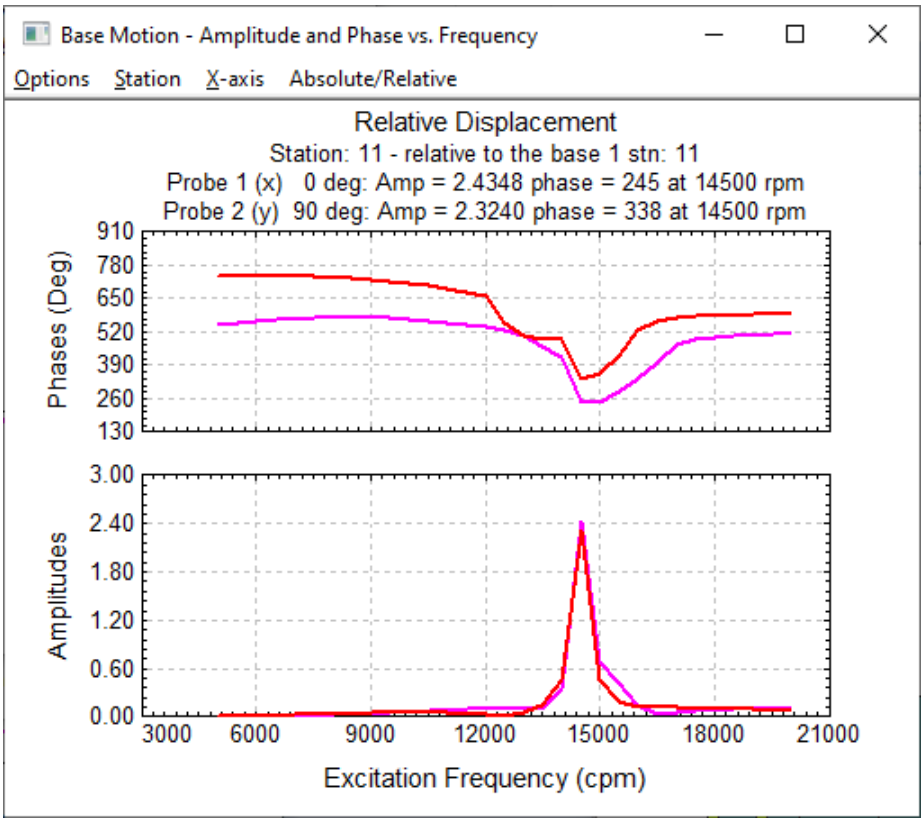

Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 6, Sub-Station: 1
Probe 1 (x) 0 deg: Amp = 2.2148 phase = 245 at 14500 rpm
Probe 2 (y) 90 deg: Amp = 2.4729 phase = 342 at 14500 rpm
Phases (Deg)
640
560
480
400
320
240
160
Amplitudes
3.00
2.40
1.80
1.20
0.60
0.00
3000
6000
9000
12000
15000
18000
21000
Excitation Frequency (cpm)

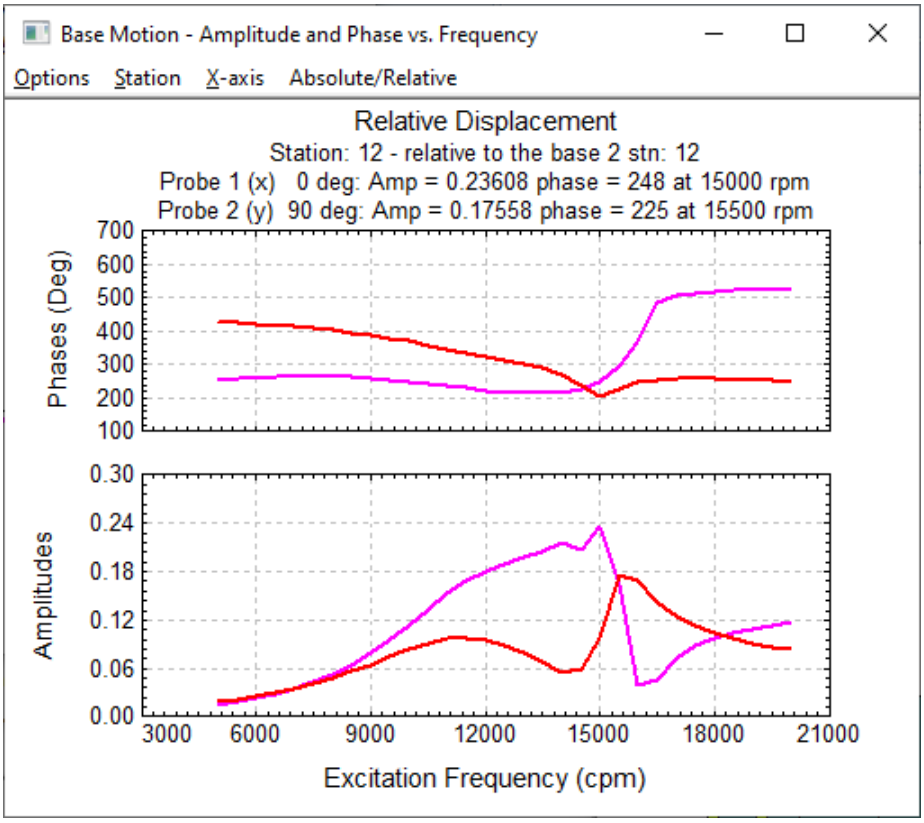

Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 7, Sub-Station: 1
Probe 1 (x) 0 deg: Amp = 0.24964 phase = 54 at 16000 rpm
Probe 2 (y) 90 deg: Amp = 0.30799 phase = 19 at 10500 rpm
Phases (Deg)
600
480
360
240
120
0
Amplitudes
0.40
0.32
0.24
0.16
0.08
0.00
3000
6000
9000
12000
15000
18000
21000
Excitation Frequency (cpm)


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 10, Sub-Station: 1
Probe 1 (x) 0 deg: Amp = 3.5587 phase = 76 at 14500 rpm
Probe 2 (y) 90 deg: Amp = 3.0120 phase = 147 at 14500 rpm
Phases (Deg)
650
520
390
260
130
0
Amplitudes
4.00
3.20
2.40
1.60
0.80
0.00
3000
6000
9000
12000
15000
18000
21000
Excitation Frequency (cpm)


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 11
Probe 1 (x) 0 deg: Amp = 2.2112 phase = 245 at 14500 rpm
Probe 2 (y) 90 deg: Amp = 2.4766 phase = 342 at 14500 rpm
Phases (Deg)
640
560
480
400
320
240
160
Amplitudes
3.00
2.40
1.80
1.20
0.60
0.00
3000
6000
9000
12000
15000
18000
21000
Excitation Frequency (cpm)


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Absolute Displacement
Station: 12
Probe 1 (x) 0 deg: Amp = 0.24764 phase = 73 at 16500 rpm
Probe 2 (y) 90 deg: Amp = 0.30750 phase = 19 at 10500 rpm
Phases (Deg)
600
480
360
240
120
0
Amplitudes
0.40
0.32
0.24
0.16
0.08
0.00
3000
6000
9000
12000
15000
18000
21000
Excitation Frequency (cpm)


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Relative Displacement
Station: 11 - relative to the base 1 stn: 11
Probe 1 (x) 0 deg: Amp = 2.4348 phase = 245 at 14500 rpm
Probe 2 (y) 90 deg: Amp = 2.3240 phase = 338 at 14500 rpm
Phases (Deg)
910
780
650
520
390
260
130
Amplitudes
3.00
2.40
1.80
1.20
0.60
0.00
3000 6000 9000 12000 15000 18000 21000
Excitation Frequency (cpm)


Base Motion - Amplitude and Phase vs. Frequency
Options Station X-axis Absolute/Relative
Relative Displacement
Station: 12 - relative to the base 2 stn: 12
Probe 1 (x) 0 deg: Amp = 0.23608 phase = 248 at 15000 rpm
Probe 2 (y) 90 deg: Amp = 0.17558 phase = 225 at 15500 rpm
Phases (Deg)
700
600
500
400
300
200
100
Amplitudes
0.30
0.24
0.18
0.12
0.06
0.00
3000 6000 9000 12000 15000 18000 21000
Excitation Frequency (cpm)