Torsional Transient Analysis Summary For Version 23.30

The following table gives the conversions between the coordinates for the actual (physical) and equivalent (generalized) systems for a geared train. Note that ***n*** is the gear (speed) ratio. For the computational purposes, let us extend the speed ratio for every angular displacement to be n_i:

$$n_i = \frac{\Omega \text{ at station } i}{\Omega \text{ at station } 1} \qquad i = 1\ldots\text{NDOF. And } \Omega_1 \text{ is the reference shaft speed.}$$

Parameters	Actual (physical)	Equivalent (generalized)
Displacement	θ	$q = \theta / n$
Inertia	J	$n^2 J$
Stiffness	k	$n^2 k$
Damping	c	$n^2 c$
Torque	T	$Q = nT$

Table 1: Relationships between the Actual Physical and Equivalent Generalized Parameters

For the purely torsional analysis of a geared system, a positive gear ratio is commonly used since the equation of motion does not involve the gyroscopic effect. Also, the sign of ***n*** does not affect the frequency analysis (free vibration) since $\boldsymbol{n^2}$ is employed in the inertia, damping, and stiffness conversion. It only affects the forced response calculation since the equivalent torque is the multiplication of torque and gear ratio ($\boldsymbol{Q = nT}$). The sign of ***Q*** determines the characteristics of the torque. The driving torque ($\boldsymbol{Q_d = nT_d}$) and load torque ($\boldsymbol{Q_L = nT_L}$) must have the opposite sign. **In the Version 23.30, for the speed dependent transient analysis, the program will check the signs of the load torques and make necessary change if needed. However, the program will not check the direction of the torques for the time dependent transient analysis. For the time dependent transient analysis, all the torques are applied torques and the signs of the rated torques (*T*) and their associated gear ratio (*n*) determine the direction of the torque.**

For the lateral vibration and coupled lateral-torsional vibration analysis of a multiple rotor system or geared system, the sign of the speed ratio becomes critical since the gyroscopic effect plays a critical role in the lateral vibration.

There are numerous torsional vibration analysis examples in the Example folder. The file names start with "Ch_8_.....". Many of them are documented in the Chapter 8 of the book "Introduction to the Dynamics of Rotor-Bearing Systems". The analysis includes the natural frequency analysis, steady state single harmonic excitation analysis, steady state multiple harmonic excitation analysis (such as engine excitation, and reciprocating excitation), transient analysis with time dependent excitations, and transient startup analysis with speed dependent excitations.

Purely Torsional Analysis of a Geared System

Several coordinate systems can be used for the torsional vibration analysis of a geared system. Two coordinate systems used in DyRoBeS are described here in details. Others can be found in Reference [1] and not repeated here. For the purely torsional vibration analysis of a geared system, the torsional stiffness at the gear mesh is much stiffer than other torsional flexibilities of the system and thus can be considered a rigid connection. Note that for the lateral and coupled lateral-torsional vibration analysis of multiple rotor systems and/or geared systems, the coordinate system must be consistent for all the rotors.

Simplest Geared System – One Degree of Freedom System

Consider the simplest geared system as shown in Figure 1. Note that the shaft 1 rotates CCW with a speed of Ω_1 and is chosen to be the reference shaft, typically is the driving shaft. Shaft 2 rotates CW with a speed of Ω_2. The associated torque (T) and angular displacement (θ) are in the same direction as the shaft rotation..

The speed (gear) ratio of shaft 2 to shaft 1 is n, that is:

$$n = \frac{\Omega_2}{\Omega_1} \quad \text{where} \quad \Omega_1 > 0, \quad \Omega_2 > 0, \quad \text{and} \quad n > 0 \tag{1}$$

n is a positive value if two shafts rotate in the opposite directions for a geared system. This is always true for a purely torsional vibration analysis. The geometric relationship for a rigidly connected gear mesh is:

$$\theta_2 = n\theta_1 \tag{2}$$

Note that the angular displacements (θ_1 and θ_2) rotate in the opposite directions, same for the associated torques (T_1 and T_2).

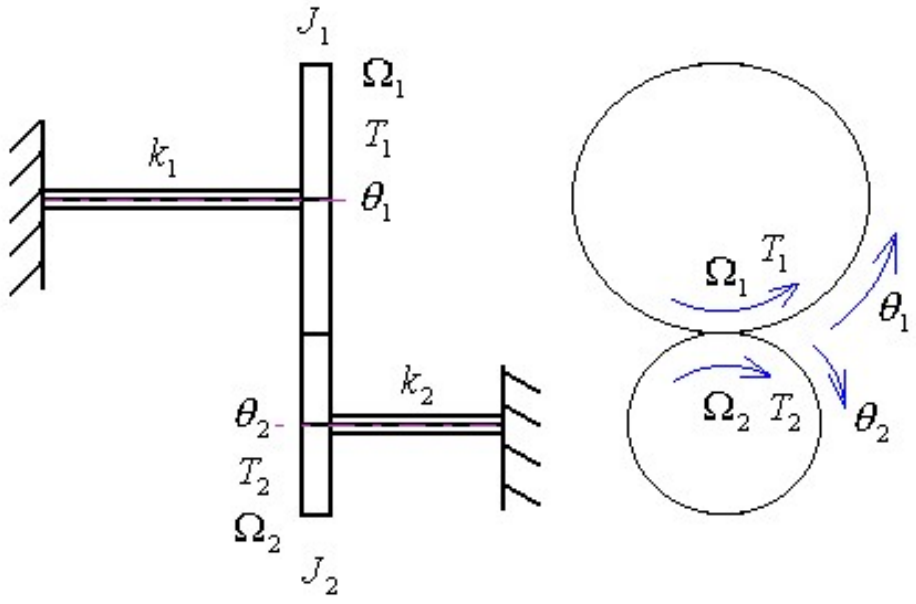

Figure 1: A Simplest Geared System – Coordinate System 1 (n>0)

The kinetic energy and potential energy of the system are:

$$KE = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 = \frac{1}{2}\left(J_1 \dot{\theta}_1^2 + n^2 J_2 \dot{\theta}_1^2 \right) = \frac{1}{2}\left(J_1 + n^2 J_2 \right) \dot{\theta}_1^2 \tag{3}$$

$$PE = \frac{1}{2} k_1 \theta_1^2 + \frac{1}{2} k_2 \theta_2^2 = \frac{1}{2}\left(k_1 \theta_1^2 + n^2 k_2 \theta_1^2 \right) = \frac{1}{2}\left(k_1 + n^2 k_2 \right) \theta_1^2 \tag{4}$$

The virtual work done by the external torques is:

$$\delta W = T_1 \delta\theta_1 + T_2 \delta\theta_2 = \left(T_1 + n T_2 \right) \delta\theta_1 \tag{5}$$

There are two localized angular coordinates (θ_1, θ_2) specified in the system, however, they are related by the gear ratio n ($\theta_2 = n\theta_1$). The total independent **generalized** (equivalent) coordinate is only one and this is a single degree-of-freedom system. Let us define the generalized coordinate q, the relationship between the generalized coordinate and physical coordinates are:

$$\theta_1 = q, \quad \text{and} \quad \theta_2 = nq \tag{6}$$

Thus, the geared system can be reduced to an equivalent single shaft system. The equivalent inertia and stiffness of shaft 2 referred to shaft 1 are $n^2 J_2$ and $n^2 k_2$. The equivalent torque is nT_2. By applying Lagrange's equation, gives the equation of motion:

$$\left(J_1 + n^2 J_2 \right) \ddot{\theta}_1 + \left(k_1 + n^2 k_2 \right) \theta_1 = \left(T_1 + n T_2 \right) \tag{7}$$

or

$$\left(J_1 + n^2 J_2 \right) \ddot{q} + \left(k_1 + n^2 k_2 \right) q = Q \tag{8}$$

The second coordinate system for a geared system, as shown in Figure 2. The shaft rotational velocity (speed) is considered to be a vector with a direction along the shaft centerline (Z-axis). The shaft rotation (Ω) is positive for the CCW rotation and is negative for the CW rotation. Then the speed ratio is a negative value for a geared system in this coordinate system. This coordinate system must be employed if lateral and coupled lateral-torsional vibrations are considered due to the gyroscopic effect involved in the lateral vibration. For more information on the lateral and coupled lateral-torsional vibration analysis, please see reference [2]. In this coordinate system, all three parameters (speed, torque, and angular displacement) are defined positive in the CCW direction along the shaft centerline Z axis (spin axis), regardless the direction of shaft rotation.

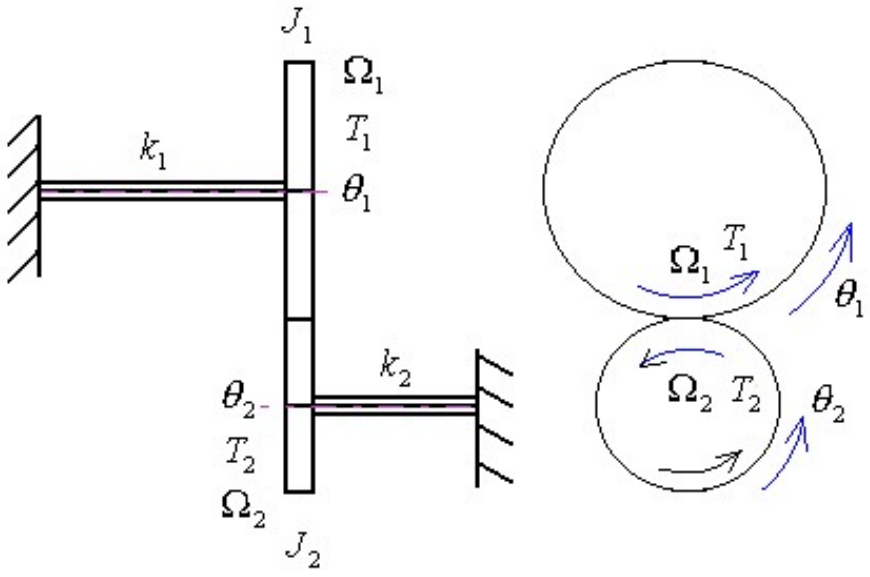

Figure 2: A Simplest Geared System – Coordinate System 2 (n<0)

For a geared system, the speed ratio and geometric constraint are:

$$n = \frac{\Omega_2}{\Omega_1} \quad \text{where} \quad \Omega_1 > 0, \quad \Omega_2 < 0, \quad \text{and} \quad n < 0 \tag{9}$$

$$\theta_2 = n\theta_1 \tag{10}$$

The kinetic energy and potential energy of the system are:

$$KE = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 = \frac{1}{2}\left(J_1\dot{\theta}_1^2 + n^2J_2\dot{\theta}_1^2\right) = \frac{1}{2}\left(J_1 + n^2J_2\right)\dot{\theta}_1^2 \tag{11}$$

$$PE = \frac{1}{2}k_1\theta_1^2 + \frac{1}{2}k_2\theta_2^2 = \frac{1}{2}\left(k_1\theta_1^2 + n^2k_2\theta_1^2\right) = \frac{1}{2}\left(k_1 + n^2k_2\right)\theta_1^2 \tag{12}$$

The virtual work done by the external torques is:

$$\delta W = T_1\delta\theta_1 + T_2\delta\theta_2 = \left(T_1 + nT_2\right)\delta\theta_1 \tag{13}$$

Again, let the generalized coordinate $q = \theta_1$, the equation of motion is identical with the previous coordinate system. Actually all the relationships and equations are the same with the previous discussion. The only difference is the speed ratio n is positive for the coordinate system 1 and negative for the coordinate system 2. The governing equations are the same in the generalized coordinates, but the generalized torque is dependent upon the torque and direction of rotation (or speed ratio). Caution should be taken while applying and converting the physical torque to the generalized torque and vice versa.

The following table gives the conversions between the coordinates for the actual (physical) and equivalent (generalized) systems for a geared train. The equivalent inertia and stiffness of shaft 2 referred to shaft 1 are $n^2 J_2$ and $n^2 k_2$, same for the damping.

Parameters	Actual (physical)	Equivalent (generalized)
Displacement	θ	$q = \theta / n$
Inertia	J	$n^2 J$
Stiffness	k	$n^2 k$
Damping	c	$n^2 c$
Torque	T	$Q = nT$

Table 1: Relationships between the Actual Physical and Equivalent Generalized Parameters

The coordinate systems are summarized below:

1) $n > 0$: The speed (Ω), angular displacement (θ), and external torque (T) are positive in the direction of shaft rotation for a geared system. The angular displacement and torque for each shaft are dependent on its own direction of rotation. For a geared system, two shafts rotate in the opposite direction, and so for their associated displacement and torque.

2) $n < 0$: The speed (Ω), angular displacement (θ), and external torque (T) are positive in the direction of shaft one rotation (reference shaft) for a geared system. Two shafts rotate in the opposite direction. The angular displacement and torque for each shaft are dependent on the shaft one (reference shaft) rotation and can be considered to be the global coordinate system along the shaft spinning axis and regardless the direction of rotation.

3. Regardless of which coordinate system is chosen, the generalized inertia, damping, and stiffness matrices are identical for the purely torsional vibration analysis. This implies that the free vibration (Eigenvalue problem) is also identical. However, the generalized torque Q is the speed ratio n multiplying the physical torque T. In this case, assume T_1 is the driving torque and T_2 is the load (resistant) torque. "Load" torque indicates that the generalized torque $Q_L = nT_L$ is negative (or in the opposite direction of the motor driving torque Q_m.

A Simple Gear Train – 3 Degrees of Freedom System

Consider a practical and simple gear train system, as shown in Figure 3, to illustrate the above discussion on the purely torsional vibration of a geared system. The system consists of a motor (J_1 or J_m) with a driving torque of T_1 (or T_m), a gear (J_2), a pinion (J_3), and a compressor (J_4 or J_c) with a load (resistant) torque of T_4 (or T_c). The speed ratio (compressor speed/motor speed) is n. The torsional stiffness between station 1 (motor) and station 2 (driving gear) is k_1, and the torsional stiffness between station 3 (driven pinion) and station 4 (compressor) is k_2. This example has been employed to illustrate the coupled lateral-torsional vibration analysis in Reference [2]. However, we only consider the torsional vibration here.

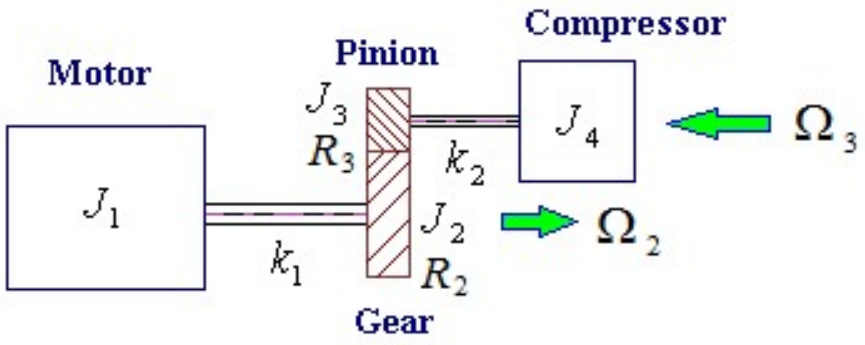

Figure 3: A Simple geared system – 3 DOFS

The kinetic energy and potential energy can be expressed in the matrix form:

$$KE = \frac{1}{2}\left(J_1\dot{\theta}_1^2 + J_2\dot{\theta}_2^2 + J_3\dot{\theta}_3^2 + J_4\dot{\theta}_4^2\right) = \frac{1}{2}\dot{\boldsymbol{\theta}}^T \boldsymbol{J}\dot{\boldsymbol{\theta}} \tag{14}$$

$$PE = \frac{1}{2}\left[k_1\left(\theta_2 - \theta_1\right)^2 + k_2\left(\theta_4 - \theta_3\right)^2\right] = \frac{1}{2}\boldsymbol{\theta}^T \boldsymbol{K}\boldsymbol{\theta} \tag{15}$$

The virtual work done by the external torques is:

$$\delta W = T_1\delta\theta_1 + T_2\delta\theta_2 + T_3\delta\theta_3 + T_4\delta\theta_4 = \delta\boldsymbol{\theta}^T \boldsymbol{T} \tag{16}$$

Where the displacement vector $\boldsymbol{\theta}$, external torque vector $\boldsymbol{T}$, inertia matrix $\boldsymbol{J}$, and stiffness matrix $\boldsymbol{K}$ are:

$$\boldsymbol{\theta} = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix} \qquad \boldsymbol{T} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} \tag{17}$$

$$\boldsymbol{J} = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix} \tag{18}$$

$$\boldsymbol{K} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 & 0 & 0 \\ 0 & 0 & k_2 & -k_2 \\ 0 & 0 & -k_2 & k_2 \end{bmatrix} \tag{19}$$

The three independent generalized coordinates (q_1, q_2, q_3) can be defined by the following relationship with the four physical coordinates $(\theta_1, \theta_2, \theta_3, \theta_4)$:

$$q_1 = \theta_1, \quad q_2 = \theta_2, \quad q_3 = \theta_4/n \tag{20}$$

or

$$\theta_1 = q_1, \quad \theta_2 = q_2, \quad \theta_3 = nq_2, \qquad \theta_4 = nq_3 \tag{21}$$

For the computational purposes, let us extend the speed ratio for every angular displacement to be n_i:

$$n_i = \frac{\Omega \text{ at station } i}{\Omega \text{ at station 1}} \qquad i = 1,2,3,4 \tag{22}$$

In this example, $n_1 = n_2 = 1, \quad \text{and} \quad n_3 = n_4 = n$

To convert the energy expressions from the physical coordinates to the generalized coordinates, let us introduce the transformation:

$$\boldsymbol{\theta} = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix}_{(4x1)} = \begin{bmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & n_3 & 0 \\ 0 & 0 & n_4 \end{bmatrix}_{(4x3)} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}_{(3x1)} = \hat{\boldsymbol{n}}\boldsymbol{q} \tag{23}$$

Then we have:

$$KE = \frac{1}{2}\dot{\boldsymbol{\theta}}^T \boldsymbol{J}\dot{\boldsymbol{\theta}} = \frac{1}{2}\dot{\boldsymbol{q}}^T \left(\hat{\boldsymbol{n}}^T \boldsymbol{J}\hat{\boldsymbol{n}}\right)\dot{\boldsymbol{q}} = \frac{1}{2}\dot{\boldsymbol{q}}^T \left(\boldsymbol{J}_q\right)\dot{\boldsymbol{q}} \tag{24}$$

$$PE = \frac{1}{2}\boldsymbol{\theta}^T \boldsymbol{K}\boldsymbol{\theta} = \frac{1}{2}\boldsymbol{q}^T \left(\hat{\boldsymbol{n}}^T \boldsymbol{K}\hat{\boldsymbol{n}}\right)\boldsymbol{q} = \frac{1}{2}\boldsymbol{q}^T \left(\boldsymbol{K}_q\right)\boldsymbol{q} \tag{25}$$

$$\delta W = \delta\boldsymbol{\theta}^T \boldsymbol{T} = \delta\boldsymbol{q}^T \left(\hat{\boldsymbol{n}}^T \boldsymbol{T}\right) = \delta\boldsymbol{q}^T \left(\boldsymbol{Q}\right) \tag{26}$$

The generalized inertia and stiffness matrices and torque vectors are:

$$\boldsymbol{J}_q = \hat{\boldsymbol{n}}^T \boldsymbol{J}\hat{\boldsymbol{n}} = \begin{bmatrix} n_1 & 0 & 0 & 0 \\ 0 & n_2 & n_3 & 0 \\ 0 & 0 & 0 & n_4 \end{bmatrix}_{(3x4)} \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix}_{(4x4)} \begin{bmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & n_3 & 0 \\ 0 & 0 & n_4 \end{bmatrix}_{(4x3)}$$

$$\boldsymbol{J}_q = \begin{bmatrix} n_1^2 J_1 & 0 & 0 \\ 0 & n_2^2 J_2 + n_3^2 J_3 & 0 \\ 0 & 0 & n_4^2 J_4 \end{bmatrix} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 + n^2 J_3 & 0 \\ 0 & 0 & n^2 J_4 \end{bmatrix} \tag{27}$$

$$\boldsymbol{K}_q = \hat{\boldsymbol{n}}^T \boldsymbol{K} \hat{\boldsymbol{n}} = \begin{bmatrix} n_1 & 0 & 0 & 0 \\ 0 & n_2 & n_3 & 0 \\ 0 & 0 & 0 & n_4 \end{bmatrix}_{(3\text{x}4)} \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 & 0 & 0 \\ 0 & 0 & k_2 & -k_2 \\ 0 & 0 & -k_2 & k_2 \end{bmatrix}_{(4\text{x}4)} \begin{bmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & n_3 & 0 \\ 0 & 0 & n_4 \end{bmatrix}_{(4\text{x}3)}$$

$$\boldsymbol{K}_q = \begin{bmatrix} n_1^2 k_1 & -n_1 n_2 k_1 & 0 \\ -n_1 n_2 k_1 & n_2^2 k_1 + n_3^2 k_2 & -n_3 n_4 k_2 \\ 0 & -n_3 n_4 k_2 & n_4^2 k_2 \end{bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + n^2 k_2 & -n^2 k_2 \\ 0 & -n^2 k_2 & n^2 k_2 \end{bmatrix} \tag{28}$$

$$\boldsymbol{Q} = \hat{\boldsymbol{n}}^T \boldsymbol{T} = \begin{bmatrix} n_1 & 0 & 0 & 0 \\ 0 & n_2 & n_3 & 0 \\ 0 & 0 & 0 & n_4 \end{bmatrix}_{(3\text{x}4)} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix}_{(4\text{x}1)} = \begin{Bmatrix} n_1 T_1 \\ n_2 T_2 + n_3 T_3 \\ n_4 T_4 \end{Bmatrix} = \begin{Bmatrix} T_1 \\ T_2 + n T_3 \\ n T_4 \end{Bmatrix} \tag{29}$$

The equations of motion for the complete gear train can be derived from the Lagrange's equations as follows:

$$\begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 + n^2 J_3 & 0 \\ 0 & 0 & n^2 J_4 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + n^2 k_2 & -n^2 k_2 \\ 0 & -n^2 k_2 & n^2 k_2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} T_1 \\ T_2 + n T_3 \\ n T_4 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} \tag{30}$$

This equation is identical to the References [1] and [2].

System Equations of Motion

The system equations of motion that describe the behavior of the entire system are obtained by assembling the elemental matrices and vectors. The element coordinates (θ) must be transformed to the generalized global coordinates (q) and the displacement continuity must be enforced in the assembly process. For a geared system, the equations of motion of each branch should be referenced to shaft 1 (the reference shaft) by multiplying the inertia, damping, and stiffness matrices by n^2 and multiplying the torque vector by n, where n is the speed ratio of the branch rotor to the reference rotor.

The assembled system equations of motion are as follows:

$$\boldsymbol{J}_q\ddot{\boldsymbol{q}} + \boldsymbol{C}_q\dot{\boldsymbol{q}} + \boldsymbol{K}_q\boldsymbol{q} = \boldsymbol{Q} \tag{31}$$

where $\boldsymbol{q}$ is the generalized coordinates referenced to shaft 1 (the reference shaft) and $\boldsymbol{Q}$ is the generalized torque vector. Frequently, the subscript q for the inertia, damping, and stiffness matrices are dropped for convenience without any confusion.

Example Verification

A compressor driven by a synchronous motor (5500 Hp) through a gear box, as shown in Figure 3, is used to illustrate the inputs for the purely torsional vibration analysis. $\boldsymbol{J}$ is the station inertia, $\boldsymbol{B}$ is the station damping connecting to the ground due to bearing loss, $\boldsymbol{K}$ and $\boldsymbol{C}$ are the element stiffness and damping connecting two inter-stations. The motor rated speed is 1200 rpm and rated torque is 288,866 Lb-in. The equivalent (generalized) 3 station model was presented and analyzed by Jung and Vance (Rotordynamics of Turbomachinery, Page 76). In order to illustrate the inputs for the gear system, let us assume the gear ratio is 2. The inputs for the original 4-station gear system and 3-station equivalent system, as shown in Figure 4, are:

$$\boldsymbol{J} = \begin{Bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{Bmatrix} = \begin{Bmatrix} 4192 \\ 2507 \\ 600 \\ 2580.5 \end{Bmatrix} Lb-in-\sec^2 \qquad \boldsymbol{B} = \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{Bmatrix} = \begin{Bmatrix} 22.99 \\ 6.99 \\ 4 \\ 5.7475 \end{Bmatrix} Lb-in-\sec$$

$$K_{12} = 73.59\ E06 \quad Lb-in/rad \qquad C_{12} = 16,663 \quad Lb-in-\sec$$
$$K_{34} = 87.925\ E06 \quad Lb-in/rad \qquad C_{34} = 9,852.75 \quad Lb-in-\sec$$

The equivalent (generalized) parameters:

$$\begin{Bmatrix} J_1 \\ J_2 \\ J_3 \end{Bmatrix}_q = \begin{Bmatrix} J_1 \\ J_2 + n^2 J_3 \\ n^2 J_4 \end{Bmatrix} = \begin{Bmatrix} 4192 \\ 4907 \\ 10322 \end{Bmatrix} \qquad \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix}_q = \begin{Bmatrix} B_1 \\ B_2 + n^2 B_3 \\ n^2 B_4 \end{Bmatrix} = \begin{Bmatrix} 22.99 \\ 22.99 \\ 22.99 \end{Bmatrix}$$

$$K_{12} = 73.59\ E06 \quad Lb-in/rad \qquad C_{12} = 16,663 \quad Lb-in-\sec$$
$$K_{23} = 351.7\ E06 \quad Lb-in/rad \qquad C_{23} = 39,411 \quad Lb-in-\sec$$

The synchronous motor excitation torque contains the average torque and oscillating torque. The compressor contains a load (resistant) torque. The excitation torque and load torque are commonly provided as percentages of the rated torque, as shown in Figure 5.

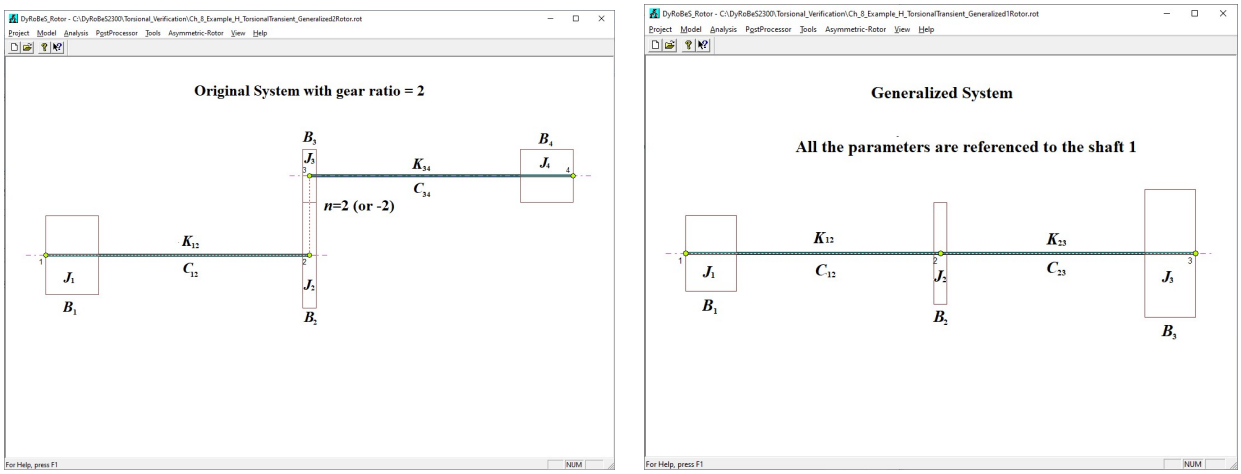

Figure 4: Example

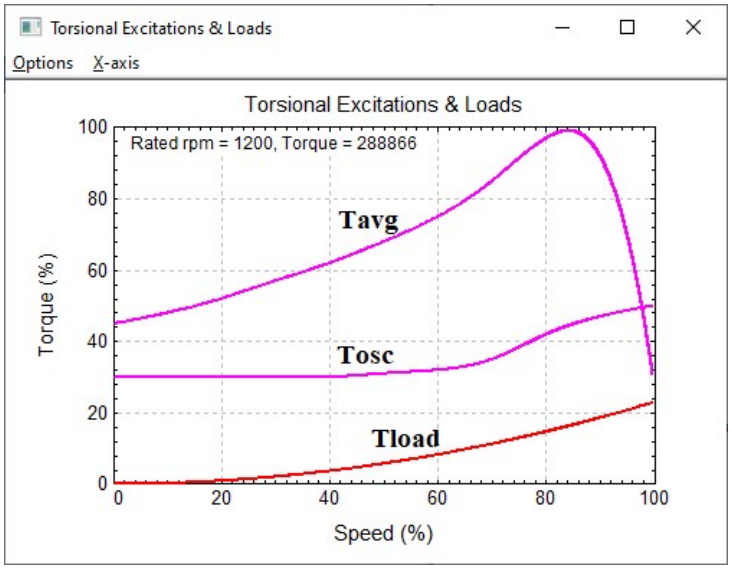

Figure 5: Motor Torque and Compressor Load Torque vs. Speed

Since the generalized (equivalent) system was presented before, we start with this 3-station equivalent system as a baseline for comparison purposes. Then we analyze the systems with the original 4-station physical parameters with speed ratio of 2 and -2 to demonstrate the correct torque inputs. The file names for this example are:

Ch_8_Example_H_TorsionalTransient_Generalized1Rotor.rot

Ch_8_Example_H_TorsionalTransient_Generalized2Rotor(n=2).rot

Ch_8_Example_H_TorsionalTransient_Generalized2Rotor(n=-2).rot

3-Station Equivalent System (Ch_8_Example_H_TorsionalTransient_Generalized1Rotor.rot)

The two undamped (damped) natural frequencies calculated from the free vibration (eigenvalue) analysis are 22.52 (22.51) and 54.74 (54.72) Hz, and the associated undamped mode shapes are shown in Figure 6.

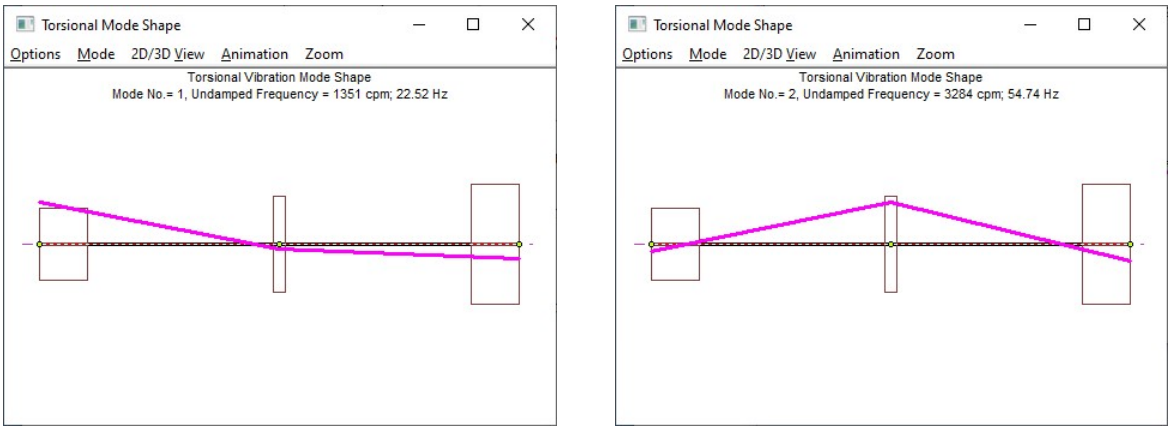

Figure 6: Undamped Mode Shapes

Two torsional Transient Analyses are available in DyRoBeS. One is the startup (speed dependent) transient analysis where the driving torque and load torque are speed dependent, the other is the time transient analysis where the applied torques, which can entered in equation form or from data file, are time dependent, as shown in Figure 7.

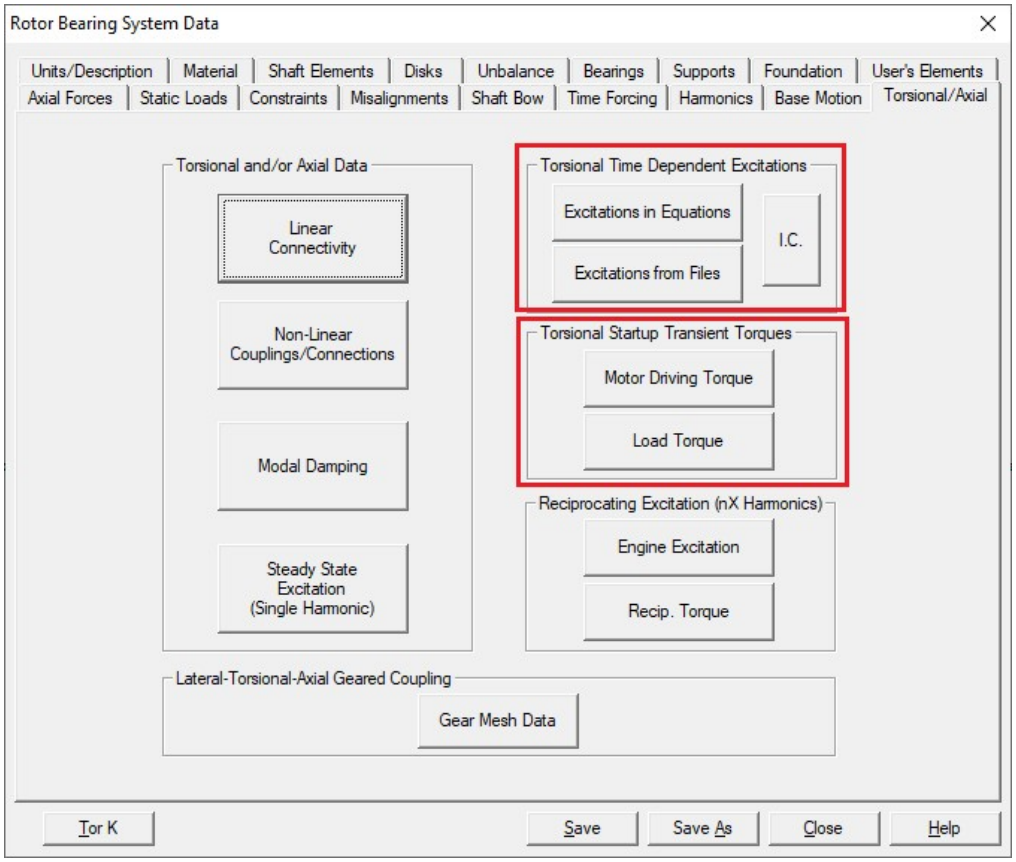

Figure 7: Torsional Transient Analysis Input Boxes

1. Startup Analysis

In this example, a synchronous motor is the driving unit. The average torque and oscillating torque are provided as percentages of the motor speed, as shown in Figures 5 and 8. The load torque is due to the compressor load as shown in Figures 5 and 9. Note that for this equivalent system (everything is referenced to the shaft 1), the compressor load torque is the converted equivalent torque, i.e. rated torque = compressor true torque X the gear ratio (Q=nT). It is customary that a positive value is provided for the load torque. Therefore, a negative sign will be added inside the program to form the "load" torque which will resist the driving torque. Let Q_m be the generalized motor driving torque and Q_L be the generalized load torque. Then they should have the opposite sign. One is positive and the other is negative. If shaft 1 is the driving shaft, then $Q_m > 0$ and $Q_L < 0$.

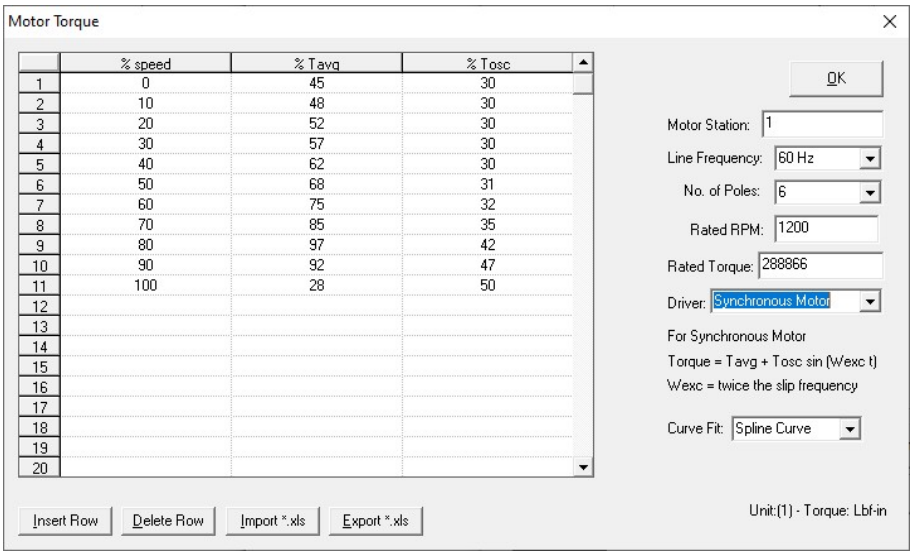

Figure 8: Motor Excitation Torque

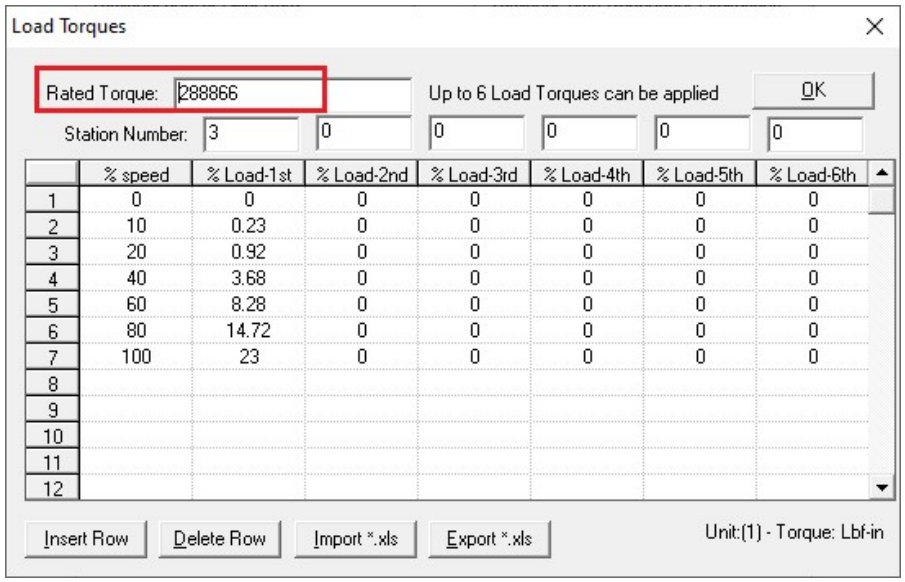

Figure 9: Compressor Load Torque

Following are the startup speed vs. time and the vibratory torques for the element 1 (shaft 1) and element 2 (shaft 2). The results are in agreement with the previous publication. Extremely small difference in results is due to the motor and load torque values obtained by digitizing the curve data from the published curves. Also, a spline interpolation was used in DyRoBeS to interpolate the data. Note that the vibratory torque for the element 2 (shaft 2) is the equivalent torque, not the true physical torque ($T=Q/n$). To convert the equivalent torque to the physical toque, go to Options – Settings, and enter 1/2 (0.5) in the Amplitude Scale since this is an equivalent single shaft system and the gear ratio is unknown to the peogram. If the system is a 4-station physical model, then simply go to Options – Settings, and select either True or Equivalent coordinate to display the requested torque.

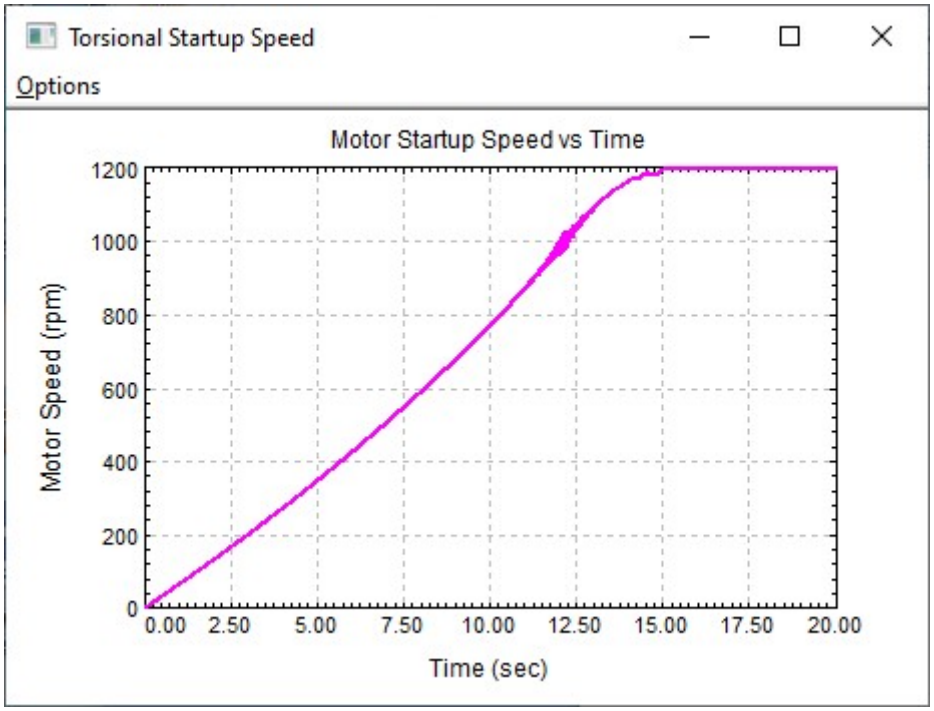

Figure 10: Motor Speed vs. Time

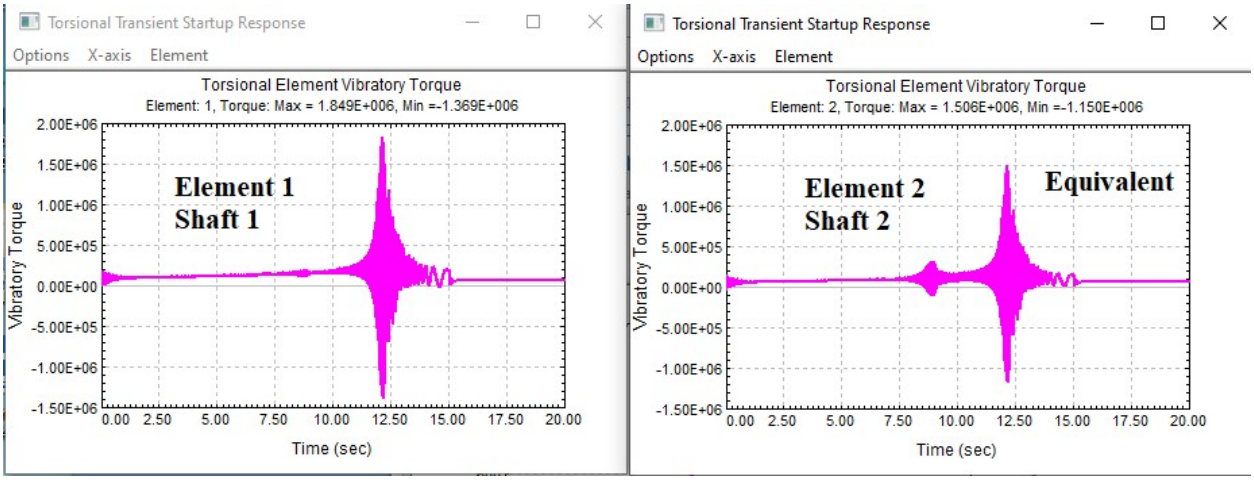

Figure 11: Element Vibratory Torques

2. Time Dependent Transient Analysis

Typically this option is for the short-circuit analysis. The applied torque can be entered using the equation form or by reading the data from external data file. Note that for the time dependent transient analysis, all the torques are "applied" torques and are treated as the excitation torques, the program will not alter the signs.

2.1) Apply a constant torque with a value of 19,421 Lb-in at station 1, as shown in Figure 12.

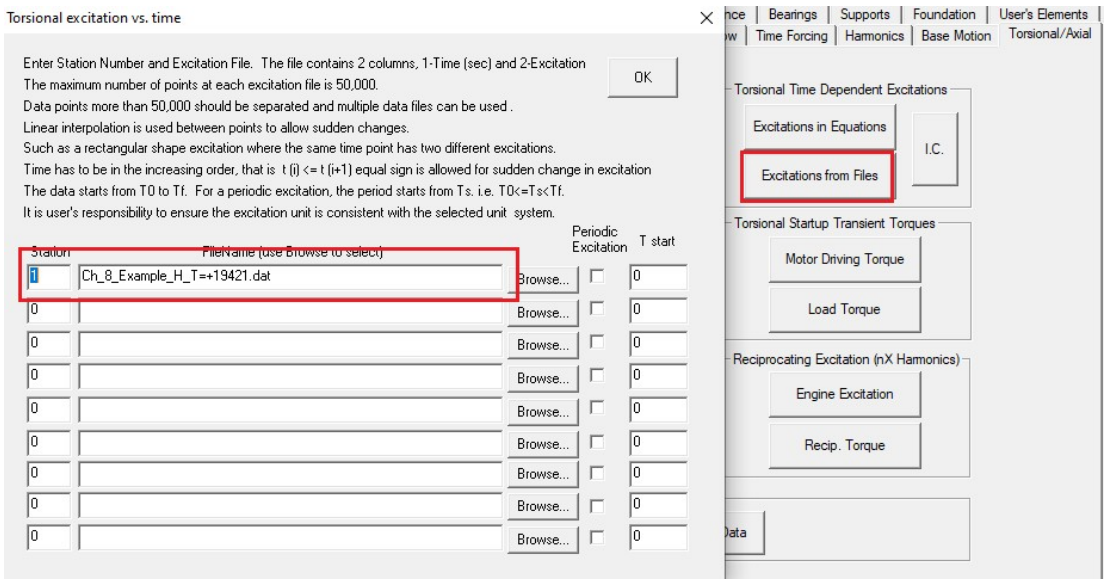

Figure 12: Apply Torque for the Time Transient Analysis

The system total inertia for this equivalent single shaft system is ($\boldsymbol{J}_1$+ $\boldsymbol{J}_2$+ $\boldsymbol{J}_3$) 19,421 Lb-in-sec^2. The applied torque is 19,421 Lb-in. If the bearing loss damping $\boldsymbol{B}$ does not exist, the acceleration will be 1 rad/sec^2. The velocity and displacement can be obtained by integrating the acceleration with time. With the bearing loss damping, the acceleration is slightly less than 1 (0.99118). The velocity and displacement at station 1 are shown in Figure 13.

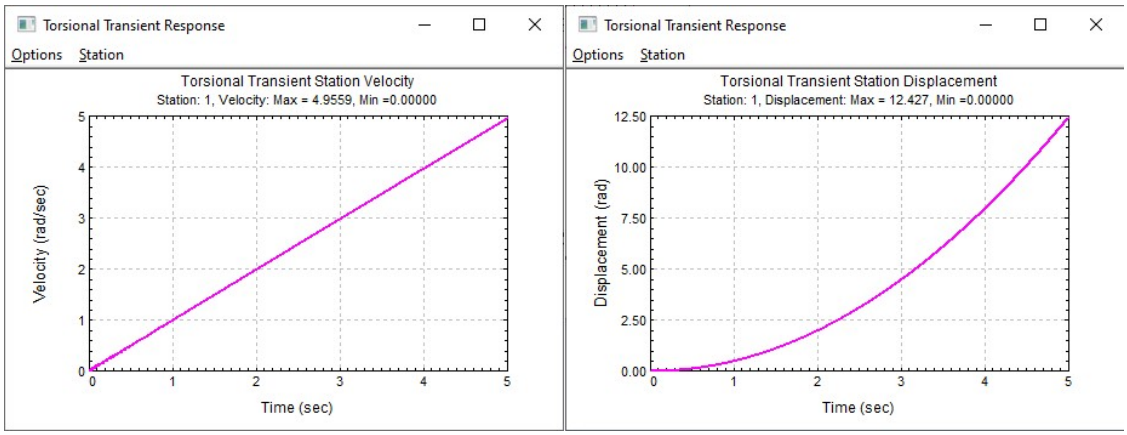

Figure 13: Velocity and Displacement at Station 1 with a Single Apply Torque

The vibratory torques for element 1 (motor shaft) and element 2 (compressor shaft) are shown in Figure 14. Again the torque at element 2 is the equivalent torque (***Q***), not the true physical torque. The transient oscillating torque at the beginning is caused by the initial condition and it oscillated at the first natural frequency and damped out with time as shown in Figure 15 for the element 1 deflection .

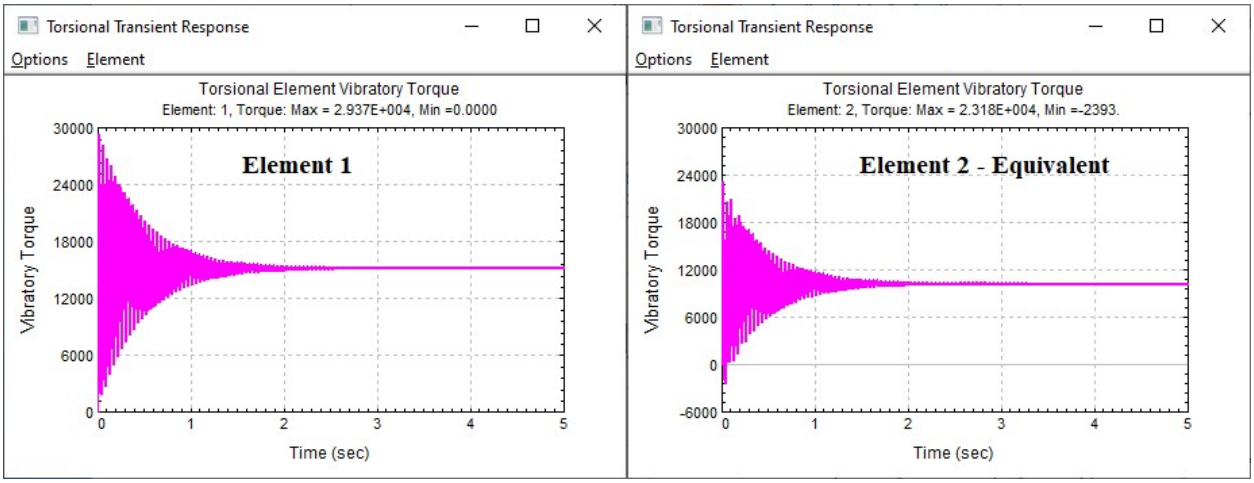

Figure 14: Vibratory Torques with a Single Apply Torque

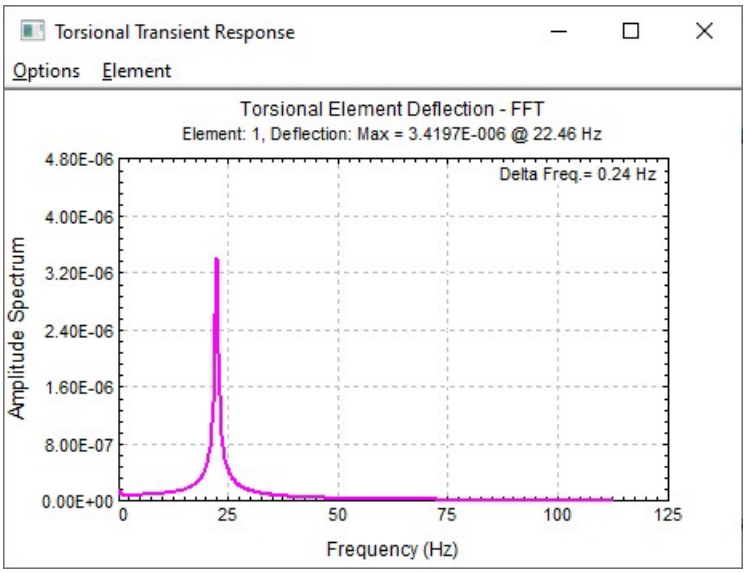

Figure 15: FFT for the element 1 Deflection with a Single Apply Torque

2.2) Let us add a resistant torque at station 3 with the same magnitude but negative value as shown in Figure 16.

The velocity, as shown in Figure 17, approaches to zero since the system reaches equilibrium after the transient motion damped out. The element vibratory torque, as shown in Figure 18, reaches to the static apply torque once the equilibrium is achieved.

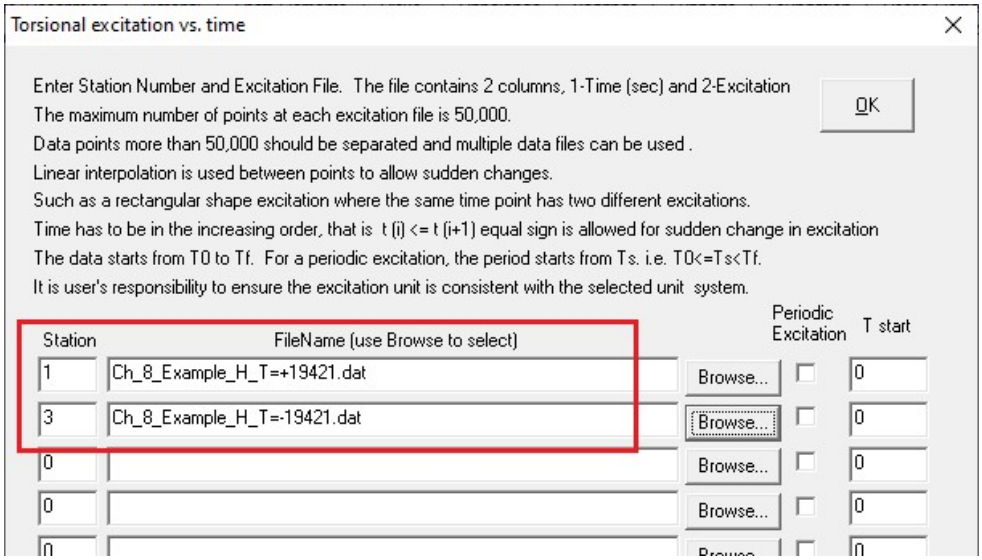

Figure 16: Apply Torques for the Time Transient Analysis

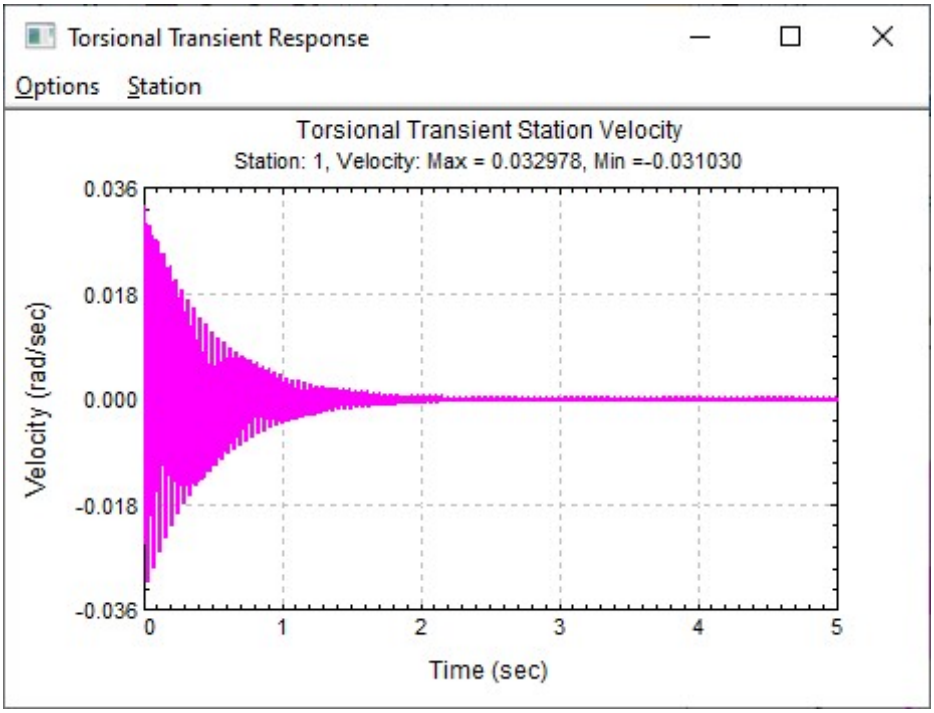

Figure 17: Velocity at Station 1 with Two Apply Torques

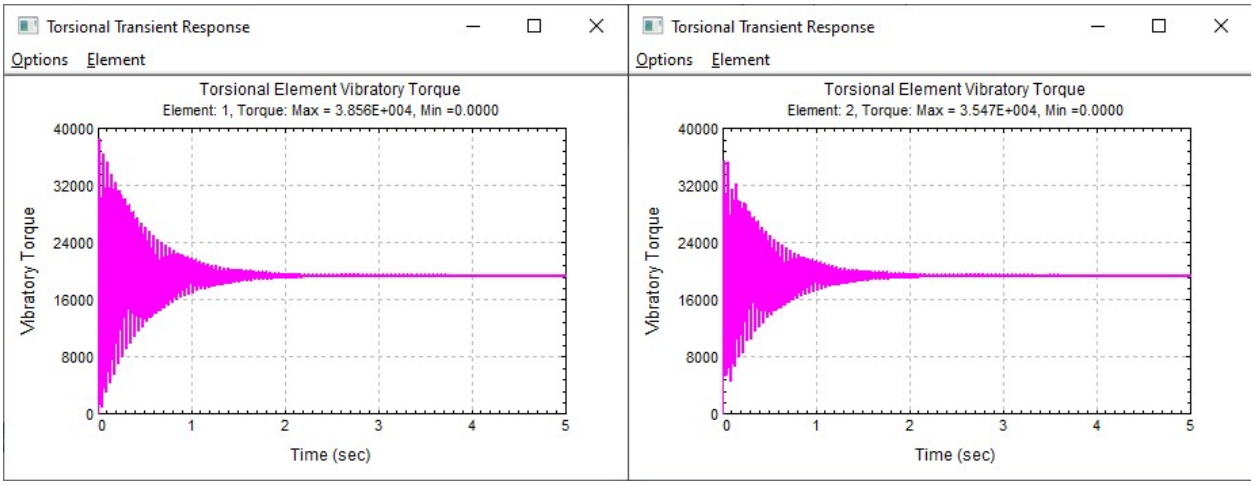

Figure 18: Vibratory Torques with Two Apply Torques

Try to replace the resistant torque at station 3 with the same torque at station 1 to see the results. The results are not presented here.

The main purpose of this document is to demonstrate the load and apply torques with positive and negative speed (gear) ratio for a geared system. We start with a positive gear ratio and then a negative gear ratio.

4-Station Two-Shaft System (Ch_8_Example_H_TorsionalTransient_Generalized2Rotor(n=2).rot)

The inputs are all in their original physical values, not converted to the reference shaft. Since the gear ratio *n*=2, the rated load torque at station 4 (shaft 2) is 288,866/2 = 144,433, as shown in Figure 19.

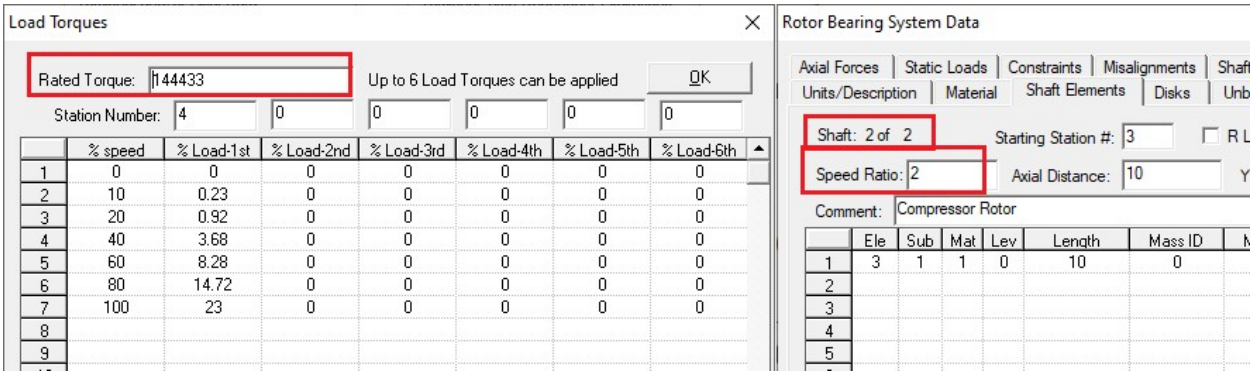

Figure 19: Compressor Load Torque for Gear Ratio = 2

The natural frequency analysis produces the same results with the equivalent 3-station system. The associated undamped mode shapes are shown in Figure 20.

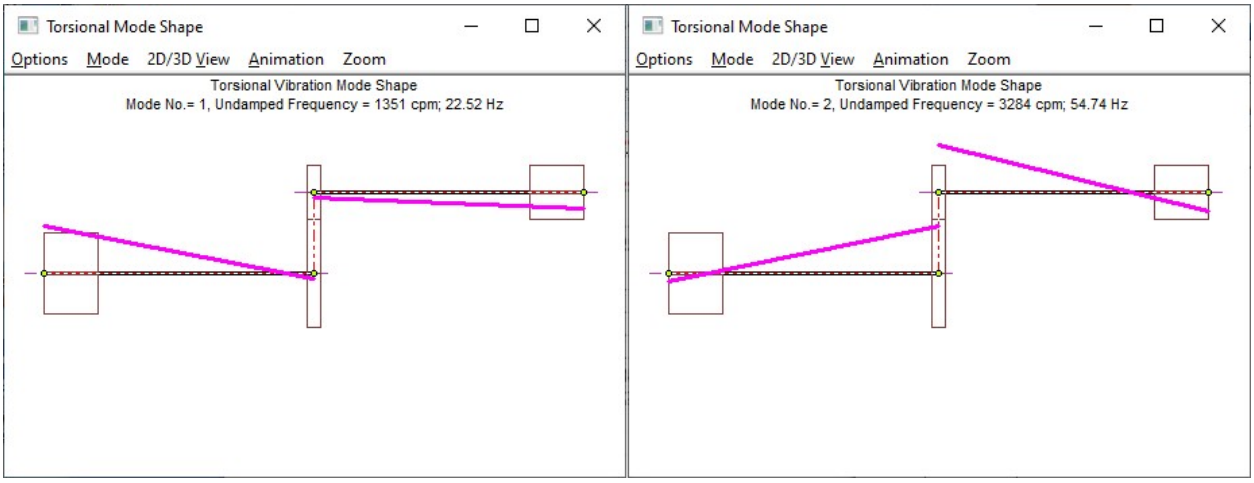

Figure 20: Undamped Mode Shapes

The startup analysis also produces the same results. The vibratory torques for the element 1 (shaft1) and element 3 (shaft 2) are shown in Figure 21. Note that the true physical torque is shown in Figure 21. To display the equivalent torque for the shaft 2, go to Options – Settings and select the Display Equivalent as shown in Figure 22. The results will be identical to the 3-station equivalent system.

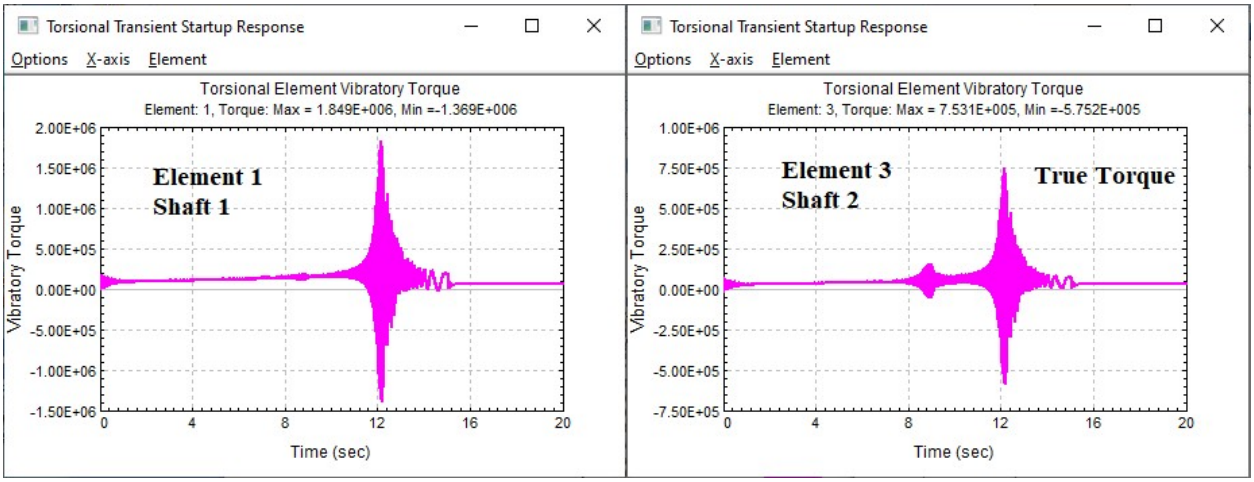

Figure 21: Element Vibratory Torques

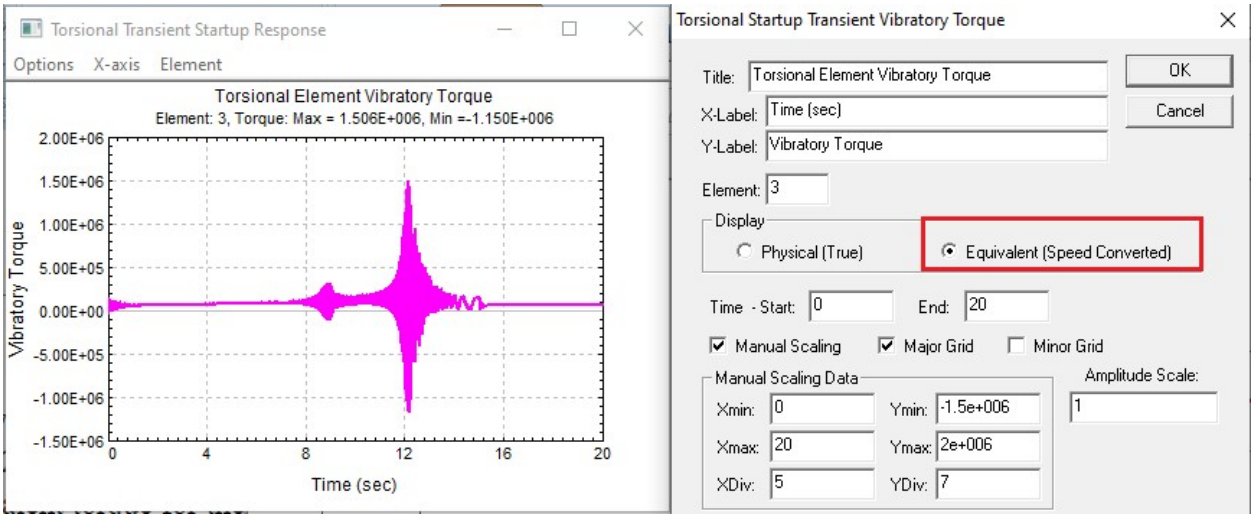

Figure 22: Element Equivalent Vibratory Torques

For the time transient analysis with a single constant torque at station 1 (motor), the results are also identical and not repeated here. To add a resistant torque at shaft 2 station 4, the relationship between the physical torque and generalized equivalent torque is $Q=nT$ or $T=Q/n$. Therefore, the physical torque T at station 4 becomes 19,421/2 as shown in Figure 23. Again, the results are identical and not repeated here.

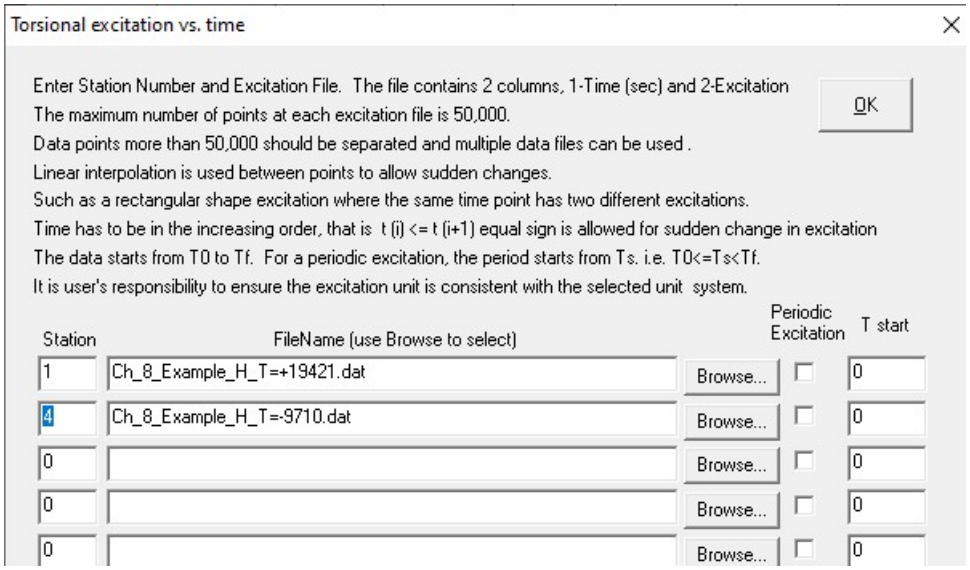

Figure 23: Apply Torques for gear ratio = 2

4-Station Two-Shaft System (Ch_8_Example_H_TorsionalTransient_Generalized2Rotor(n=-2).rot)

If we enter the speed ration = -2, the rated torque for the "Load" torque becomes negative, as shown in Figure 24. Note that in Version 23.30, the program will check the sign of the load torque and make necessary change if needed.

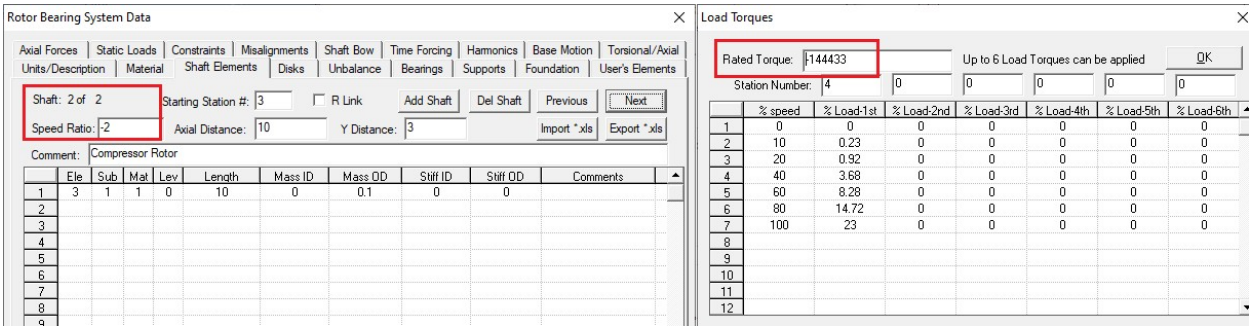

Figure 24: Load Torques for gear ratio = – 2

Again, the results are the same as before for the startup analysis. The vibratory torque for the element 3 (shaft 2) is shown in Figure 25. Note that the equivalent torque is identical to the previous results, however the physical torque has the opposite sign with positive gear ratio. It is because the torque is in opposite direction wieth positive and negative speed ratio, as illustrated in Figures 1 and 2.

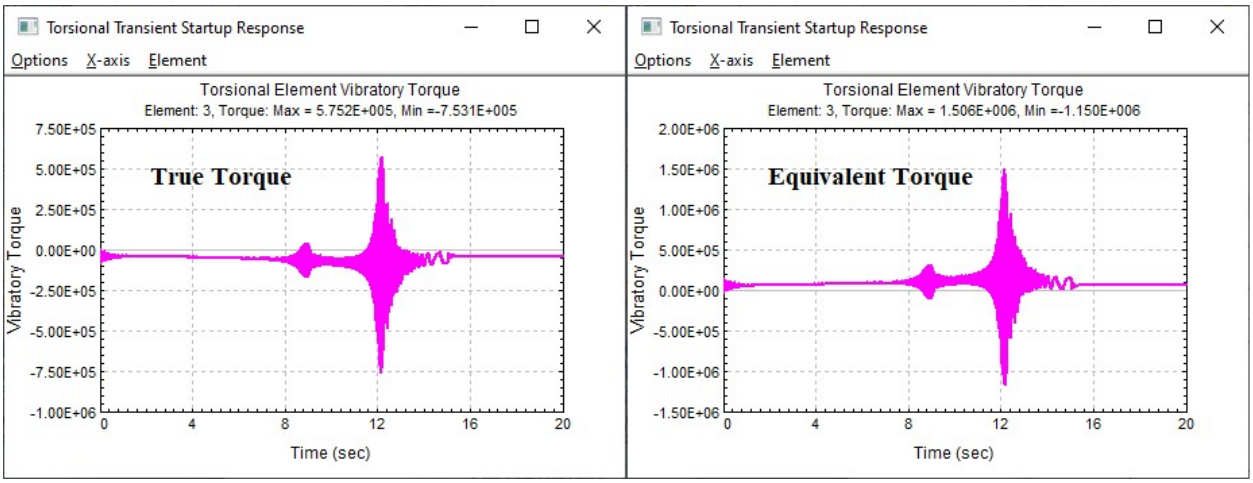

Figure 25: Vibratory Torque for Element 3 with gear ratio = – 2

Same logics applied to the time dependent transient analysis. To apply a resistant torque at station 4, a positive torque is needed due to the negative gear ratio, as illustrated in Figure 26. Again, the results are identical and not repeated here.

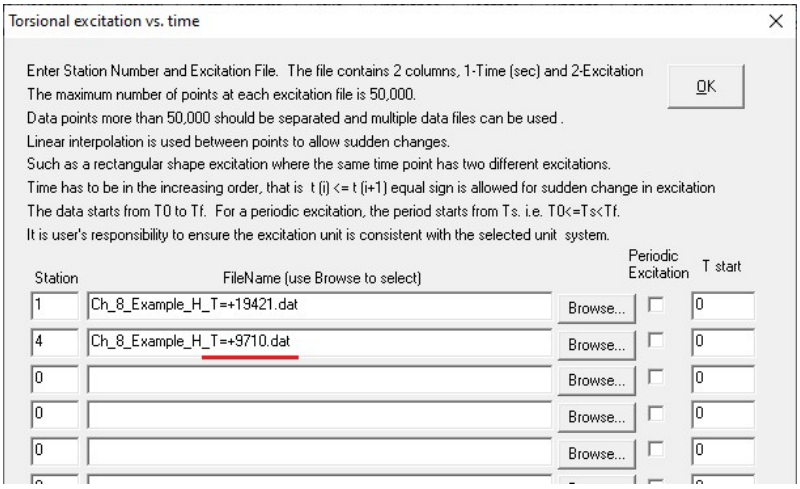

Figure 26: Apply Torques for Element 3 with gear ratio = – 2

Summary

1. The sign (positive or negative) of the speed ratio (n) determines the direction of the associated torque (T) and angular displacement (θ), as illustrated in Figures 1 and 2.

2. All the input parameters are converted into the equivalent (generalized) parameters for the analysis. Table 1 (Page 4) summarizes these relationships. The torque relationship is $\boldsymbol{Q=nT}$ and $\boldsymbol{T=Q/n}$.

3. For the startup (speed dependent) transient analysis, the rated torque (T_L) for the "Load" is positive when $n>0$ and negative when $n<0$. This is because Program will add a negative sign for the "Load". This is assumed that the rated torque (T_d) for the driving toque is positive. Otherwise, the opposite sign is applied. **Note that in the Version 23.30, the program will take care of the signs for the load torques since the equivalent torques ($Q_L=nT_L$) of the load torques must be in the opposite direction of the driving torque ($Q_d=nT_d$).**

4. For the time dependent transient analysis, all the torques are so-called applied torques. Program will treat all the torques as driving torques. Therefore, the sign of the rated torque for each torque is critical. Again, the simple rule is that $\boldsymbol{Q=nT}$ and the sign of $\boldsymbol{Q}$ determine the direction of the torque.

5. For the purely torsional analysis of a geared system, a positive gear ratio is commonly used since the equation of motion does not involve the gyroscopic effect. However, for the lateral and coupled lateral-torsional vibration analysis of a multiple rotor system or geared system, the sign of the speed ratio becomes critical since the gyroscopic effect plays a critical role in the lateral vibration.

Reference

1. W. J. Chen and E. J. Gunter "Introduction to the Dynamics of Rotor-Bearing Systems", Chapter 8: Torsional Vibration.

2. W. J. Chen "Practical Rotordynamics and Fluid Film Bearing Design", Chapter 8: Coupled Vibrations of a Geared System.

3. J. M. Vance "Rotordynamics of Turbomachinery", Page 76-83.